COMPRESSED IMAGE PRODUCTION, STORAGE, TRANSMISSION AND PROCESSING

BACKGROUND OF THE INVENTION

This invention relates to a method for producing an image of an object storing, transmitting and processing the same.

In this application, "object" means any entity that can be defined, in principle, by geometrical and/or mathematical data and/or geometrical or mathematical or empirical relationships, such as functions, correlations, regressions, lines and surfaces, etc. It is irrelevant whether the object is so complex that the number of data and/or relationships required to define it is so great that complete or exact definition is practically impossible. It is also irrelevant how many dimensions the object has. The object may be a physical one, such as a picture, a line, a surface, a solid, a tri-dimensional object or a landscape, etc., or an abstract one, such as a tensor, a form defined in a continuum having more than three dimensions, etc.; or it may be constituted by an array of data which have only a conceptual relationship with one another.

"Image" means any entity that represents an object exactly, or more or less approximately. The image may have the same nature as the object it represents, as when, e.g., it is the reproduction of a picture or an array of data representing another array of data; it may be an image in the common

meaning of the word, as when, e.g., it is a picture of a person or a landscape; or it may be quite different in nature from the object, as when, e.g., it consists of a plurality of numerical data representing a physical entity. "Intermediate image" means an image that is produced for the purpose of transforming it later into a different image of the same object, as when, e.g., a set of numbers temporarily represent a geometrical form and a geometrical image is to be developed from them. When such transformation occurs, the image finally produced will be called hereinafter "the final image". An image which is to be processed in any way elaborated to produce another image of the same nature - e.g. a first set of numbers from which another set of numbers is to be obtained, by any appropriate procedure, said other set of numbers being an intermediate or a final image, will be called a "temporary image", which, if the processing is a correction or adjustment, is an "unadjusted image".

In a great many technical processes, an image of an object must be produced, and quite often must be stored, transmitted or processed. For instance, it is a common occurrence that two-dimensional figures or pictures be represented by digital data which are stored, processed and transmitted, according to needs. This occurs in word processing by computers, message transmission by telefax, etc. Three-dimensional objects, including landscapes, may be represented by a process that is essentially the same. The representation of objects which have more than three dimensions involves in principle no conceptual departure from the said methods. Another common occurrence is the representation, storage and processing of data representing physical relationships, statistical

regressions or ways of experimental data. The use of mathematical models is also an instance of object representation by an image, which may be constituted by an array of digital data.

It is obviously desirable to reduce as much as possible the amount of data defining the image which represents a given object, without disorting the image to the extent that it might cease to represent the corresponding objer⁺ with an acceptable degree of accuracy. Such a reduction of the required data, or "data compression" or "image compression", as it is sometimes called, serves to simplify, reduce and render more economical the equipment required for the storage of an image, its processing and transmission. For instance, it is well known that in modern technology, transmission lines, including frequency bands available for radio transmission, are increasingly overcrowded, and every effort is being made to exploit them as fully as possible, one of the means for so exploiting them being to reduce the amount of data that are sent through a given transmission line in order to convey a given amount of information.

It is a general purpose of this invention to provide a method for producing the image of an object of any kind, storing it, processing and transmitting it, while minimizing the amount of data that are required for carrying out the said operations.

More specific objects of the invention and specific applications thereof, will become apparent as the description proceeds.

BRIEF SUMMARY OF THE INVENTION

The following considerations are preliminary to an understanding of the process according to the invention. If the object is defined geometrically or analytically - whether by a graphic representation or a model, depending on the nature of the object, or by an array of numerical data which are assumed to define the object or in any suitable way - it may be broken u⁻ into, viz., be considered as defined by, a plurality of components, such as lines or surfaces defined in a space which may have more than three dimensions, arrays of numerical values or functions or operators which can be represented by such lines or surfaces. For the sake of simplicity, the process according to the invention will be described firstly with a reference to an object which may be broken up into a number of plane lines, corresponding to functions of one variable. Description and definition of the process will be then expanded to those objects which must be broken up into surfaces in a three-dimensional space or in hyperspace, having more than three dimensions, corresponding to functions of two or more than two variables. Essentially the process, as described and defined, extends to compressed images of any objects that can be defined by an array of data, by software or hardware for the production and/or elaboration of digital values, such as a special purpose computer or a computer program, or by an analogical circuit or special purpose analogical computer or analogical computer program, or by digital or analogical sensors, or the like. In what follows, the term "object" will be construed as preferably meaning the physical entities and/or relationships by which the object is defined or into which the object has been translated, and which will have been stored or

memorized, as in an electronic memory, e.g. in the form of digital values or instructions relative thereto or analogical representations of functions or relationships.

In one of its simplest forms, the object, an image of which is to be constructed, may be a plane line. The object line, as any other object, may be defined in many different ways, but, for the purposes of illustration only, it will be treated as defined by a graph or by a corresponding function, being evident that the information conveyed by a graph can be conveyed in other suitable way. In any case, in order to carry out the process according to the invention, the object line must be translated into digital values or into a computer program or subroutine or an analogical process or into the structure of a special purpose digital or analogical computer, which can be entered and memorized in an elaborator, and which define couples of values x, y for each point of the line. The object line may be considered in its entirety, or, more frequently, it will be divided into segments, to each of which the process of the invention will be separately applied. Therefore, if the line has been so divided, the expression "object line", when used hereinafter, must be construed as meaning the particular segment under consideration at the moment .

The process, then, comprises, in a restrictive definition, the following steps:

(1) Approximating a line by a model which includes at least one differentiable component.

(2) Establishing the maximum allowable error ε and the degree k of the Taylor polynomials by which the differentiable component(s) of the model are to be approximated.

(3) Establishing at least a pitch grid h and constructing a grid each region of which has one of said pitches h.

(4) Computing the coefficients of the Taylor polynomials of the aforesaid differentiable component or components at selected points of said grid.

Two or more of the aforesaid steps may be carried out concurrently, in whole or in part, or divided into successive stages, which may be intercalated to a greater or a lesser extent.

Further operations, hereinafter described, may be carried out and are often desirable to minimize the effect of inaccuracies in the said coefficients, for rounding them off, for taking into account different scales which may be present in the data, and for obtaining, if desired, an image which has the same nature as the object. "Non-differentiable component" means herein a component comprising one or more points at which it is not differentiable, or, a component that is not differentiable at all its points.

The process according to the invention can be extended to objects that are more complex than plane lines by simple generalizations, as will be explained hereinafter.

BRIEF DESCRIPTION OF THE DRAWINGS

The invention will be better understood from the following description of preferred embodiments, with reference to the appended drawings, where:

Figs. 1a 1nd 1b illustrate an example of an object line and its image, respectively;

Figs. 2a and 2b illustrate a temporary image line the segments of which do not match at meeting points, and a corresponding adjusted image line, respctively;

Figs. 3a, 3b, 3c, and 3d illustrate respectively an object line and the corresponding model line, final image and non-differentiable component of the model, with reference to Example 1;

Figs. 4a and 4b represent a picture and its image, respectively, with reference to Example 2;

Fig. 5 represents a processed image of the picture of Fig. 4a, with reference to Example 3; and

Figs. 6a and 6b represent the negative of the picture of Fig. 4a and its image, respectively, with reference to Example 4.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENTS

The process steps hereinbefore defined will now be more fully explained.

Step (1) - The object line, the data defining which have been physically stored e.g. in an electronic memory, is approximated by a model, preferably defined in the same way as the object line, which model preferably consists of at least a first component embodying the characteristics of the object, if any, which render it non-differentiable at some points or regions - it being of course possible to omit said first component if there are no significar characteristics of non-differentiability of the object - and at least a second component which embodies all the differentiable content of the object. Typical cases of models are the following:

Case a) The first component is a base line, which is a simple - desirably, the simplest - line having qualitatively the same discontinuities as the object line, and the second component is a curve which represents the deviations therefrom of the object line, and which will be differentiable and can be called interpolating function. The base line may be constructed in each individual instance, or, more conveniently, may be chosen, according to the actual discontinuities of the object line, from a number of norma. forms, which are the simplest functions having the required discontinuities. The following standard form of model can be used in this case:

(1) $\Phi(x) = Hx_0,a,b,c,d(x) + \phi(x)$

wherein H is a normal form defined by $H(x) = a(x-x_0) + b$, if $x \geq x_0$ or $H(x) = c(x-x_0) + d$, if x is less than x_0. The values of the parameters x_0,a,b,c,d are determined, in a preferred embodiment of the invention, by minimizing a quantity representing an error, e.g. the quadratical error, as hereinafter set forth. The base line can be predetermined, or chosen, in general

according to predermined criteria, from a list prepared in advance, or it can be chosen in each case by the operator. This case is illustrated at Fig 1a, 1b showing respectively an object line and its model.

Case b) The model is a differentiable function of another function which embodies the non-differentiable characteristics, viz the discontinuities, of the object line. It can be epressed as:

(2) $\Phi(x) = \Phi'[\phi(x)]$,

wherein ϕ is the first component , which will be called the base curve, and Φ' is the second component. $\phi(x)$ can be looked at as defining a change of coordinates: in the differentiable component Φ', the ordinates are referred to abscissae which are not x, but $\phi(x$).

Case c) This case will be mentioned here, though it is not applicable to a line, but only to surfaces in a space having three or more dimensions. In the case of three dimensions, a coordinate (say, the elevation) z of a surface, is a function z_1 in a certain region of the plane x-y of the two remaining coordinates and is another function z_2 in another region thereof, the two regions being separated by a border line defined e.g. by a relationship $y=\phi(x)$. Then the model $\Phi(z,y)$ consists of the function z_1 if y is greater than $\phi(x)$, and z_2 if y is smaller than $\phi(x)$, one or the other of the z_1 and z_2 applying when $y=\phi(x)$.

Case d) The object line is differentiable at all points, and the model consists only of a differentiable component.

In a form of the invention, all the parameters of the model the values of which have to be chosen, are determined by minimizing a quantity representing an error - e.g. the quadratical error, viz. $\Sigma[f(x_i) -\Phi(x_i)]^2$ - the

minimization being carried out by means of a predetermined subroutine with respect to all the parameters of the model Φ, for the function f(x) representing the object, the values of f(x) for each x being determined by known subroutines. Programs for this purpose are available, e.g. from the ILSM library.

Step (2) -   a) - The maximum allowable error ε, which is to be tolerated i approximating the object line, viz. which expresses the desired precision of the image, is established.

b) - The degree k of the Taylor polynomials, which will be used to approximate the differentiable component or interpolating curve, is established.

Step (3) - The grid need not be cartesian and its coordinate lines may be curved, although for simplicity's sake a cartesian grid will always be illustrated herein. The grid may be divided into different regions having different grid pitches or even different types of coordinate lines. The grid pitch h (viz., the distance between adjacent coordinate lines which define the grid cells) is selected according to the precision desired of the image, and may be different in different parts of the region, although a regular grid will often be preferred.

In an embodiment of the invention, h is calculated, by a suitable subroutine, from the formula

(3) $CM_h h^{k+1} \leq \varepsilon$

wherein $C = 1/(k+1)!$ and M is the maximum, at each grid point, of the absolute value of the (partial, in the case of an object which is a function of more than one variable) derivatives of degree k+1 of the differentiable component or components, in the segment or zone of the object under consideration, M being determined by using a known subroutine which computes the derivatives of order k+1, produced e.g. by a package such as MAXIMA OR MATHEMATICA.

Step (4) - The nodes of the grid are taken as base points, and a (known, e.g. a MAXIMA) subroutine is applied at each base point to compute the Taylor polynomials of degree k of the interpolating curve.

At this stage, the following data have been obtained:

A) The coefficients of the Taylor polynomials of the differentiable component or components of the model;

B) The number or other identification or analytical definition of the non-differentiable component(s) , if any, of the model, such as the base line or the base curve;

C) The values of the parameters of the said non-differentiable component(s), if any;

and these define an image, which will usually be an intermediate image, but could be a final one, according to cases. Hereinafter it will be assumed that it is an intermediate image, from which the final image, in the same form as the original object, is to be constructed; however this is done merely for the sake of simplicity and involves no limitation.

In many cases, as will be explained below, the image thus obtained may require further elaboration without changing its nature, viz. while remaining a set of data of the same kind, and it will be only a temporary, in particular an unadjusted image. Then some or all of the steps from (5) on will be carried out.

Step (5) - In the case of the presence of so-called noise or inaccuracies in said temporary image line, or if the numerical noise, viz. the inaccuracies of the computations, which are large in comparison with the accuracy required, the Taylor polynomials which make up the temporary image line or its differentiable component may disagree at their meeting points by more than allowed by the required accuracy, as represented, by way of example, in Fig. 2a.

In this case, an adjusted image line is constructed by applying to each differentiable component a subroutine, hereinafter "Whitney subroutine", which computes W, wherein W is a quantity representing the discrepancies of the Taylor polynomials. In particular, W can be given by the formula:

(3) $\quad W = \Sigma_{i,j} \parallel p_i - (p_j)_i \parallel^2$

Here the sum is taken over all the adjacent grid points i, j (possibly belonging to different segments of the image). p_i, p_j denote the Taylor polynomials, obtained in steps (1) - (4) at the grid points i, j, and $(p_j)_i$ denotes the polynomial p_j, expressed in coordinates, centered at the i-th grid point. $\parallel p - q \parallel^2$ denotes, for any two polynomials p and q of the same degree and number of variables, the sum of squares of the differences of corresponding coefficients.

For any values of the coefficients of p_i, W is computed by using known subroutines, produced e.g. by a package such as MATHEMATICA.

W is then minimized (e .g. by standard gradient methods), using, as starting values of the coefficient of the Taylor polynomials, those obtained by the previous steps, and under such constraints that the result of the minimization do not deviate from the initial data by more than the allowed error, e.g. under the condition that the zero degree coefficients of saī polynomials remain unchanged. An adjusted image line, corresponding to the unadjusted image line of Fig. 2a, is illustrated by way of example in Fig. 2b.

Step (6) - If the accuracy of the adjusted coefficients of the Taylor polynomials obtained from step (5) is excessive with respect to that desired in the final image, they are rounded off to a maximum allowable error ε' by any suitable method (not necessarily the same for coefficients of different degrees). The data thus obtained represent the adjusted image line.

Step (7) - Sometimes the data of the object to be represented may require the use of different grid resolutions, or such use may be desirable. An example which clarifies this case is the following.

Let us assume that the object represents a periodic phenomenon, e.g. an oscillatory phenomenon such as an oscillating eletcrical impulse or an electromagentic wave. Such a phenomenon can be analyzed and is usually represented by the combination of two or more superimposed components, specifically, a relatively low frequency carrier wave and a higher frequency modulating wave. The modulation can be sometimes considered as

resulting from a first, intermediate frequency modulation, and one or more high frequency modulation or modulations, and in this case the object will have three or more components. The image can be conveniently constructed from images of ther various components, e.g. of the carrier wave and of the modulating wave or waves, and obviously the lower frequencies will require lower resolutions and larger grid pitches will be suitable for them. Likewise, the frequency of an oscillatory phenomenon may vary at different times or in different spatial regions and its components will not be superimposed, but separated in space. Similar situations may occur in various cases. Generally, many kinds of object may comprise superimposed or separated components which have details of different fineness, which require different degrees of resolution. Since oscillatory phenomena are a typical case of objects requiring different grid resolutions, the word "frequency" will be used to indicate the fineness of the required grid, but this is not to be taken as a limitation, since the same procedure can be applied to non-oscillatory phenomena.

In such cases, the following procedure is preferably followed:

a) Steps 1 to 6 (or such among them which are necessary in the specific case) are carried out and a first temporary image is obtained.

b) A new maximum error ε_2, bigger than ε (or ε', as the case may be) is chosen.

c) A grid which is sparser than the one used for carrying out the steps under a), and the pitch of which is determined by the resolution required by the lowest frequency of the components existing in the object (e.g. that of a carrier wave) is established.

d) Steps 1 to 6 are repeated using ε_2 and the sparser grid and a second temporary image is obtained.

e) The second temporary image thus obtained is substracted from the first and a first residual image is obtained, which contains only data relating to higher frequency components of the object.

f) The same procedure - steps b) to e) - is repeated for successively higher frequencies of components, correspondingly obtaining successiv¯ residual images increasingly restricted to higher frequency components.

As a result, coefficients of Taylor polynomials are obtained on several grids having increasingly higher resolutions, viz. smaller pitches, separately corresponding to the object components requiring increasingly higher resolutions.

The data obtained after steps (1) to (4) and those among (5), (6), (7), which it has been found necessary to perform, constitute an intermediate image or someyimes a final one. Usually these are the compressed data which can be stored, transmitted and processed.

If a further compression is desirable, one of the standard methods of encoding coefficients (e.g. Hoffman coding) can be applied. If necessary, the resulting string of data can be further compressed by one of the standard methods of unstructured data compression (e.g. entropy compression). However, this last step reduces the possibility of a compressed data processing.

If a final image, which has the same nature as the object, is to be constructed, the following procedure is followed:

Step (8) - a) The Taylor polynomial coefficients obtained after completion of steps (1) to (4) and of those among steps (5) and (6) which it has been found necessary to perform, are treated as if they represented an unadjusted temporary image, which is affected by noise, and are subjected once more to step (5), using them as starting data.

b) The domain in which the temporary image has been defined is divided into regions by means of a grid, each region being a portion of the grid around a grid node or base point. These regions may overlap.

c) A curve or curves representing the Taylor polynomials of degree k in the above regions are constructed from the coefficients defining the temporary image - e.g. obtained as in step (8) a) - at each node of the grid or of that grid having the highest resolution (smallest pitch), if there are more than one grid (particularly if step (7) has been carried out), using a known subroutine.

Said curve or curves constitute the final image of the object line.

The aforementioned curves may diverge at the meeting points of the regions mentioned above under b) (or on their overlapping parts). If this disagreement does not exceed the allowable error ε, any of the overlapping curves curves can be used at the meeting points on the overlapping parts of the above regions.

If as the result of the noise of the data or the computational noise, the above discrepancies are large in comparison with the accuracy required, average values can be used on the overlapping parts. This is done by

averaging the values of the overlapping curves with the appropriate weights.

Actually, other polynomials or functions could be used for approximation purposes, such as Tchebicheff polynomials, trigonometric exponential functions, etc., without departing from the invention, but Taylor polynomials are preferred.

The above described process applies, with obvious generalization, to a wide range of objects. Some examples follow.

I - A surface in a three-dimensional space corresponds to a function of two variables. If the surface is defined in a space that has more than three, say, n+1 dimensions, the independent variables will be more than two, say, n $(x_1,x_2,...x_n)$, but the operations to be carried out will be essentially the same, and the necessary generalizations will be obvious to skilled persons. In any case, any surface can be translated, as well as a line, to digital values, which can be entered and stored. The model will be constructed in the same way as for a line. Case c) of model construction, already described, applies to surfaces in any space. Analogously to case a), a model may consists of a simple base surface, which presents the discontinuities of the object surface, and by an differentiable or interpolating surface, which represents the deviations of the object from the base surface. One can also operate analogously to case b), by using functions of more than one variable. The minimization of the quadratical error is effected in the same way as in the case of an object line, using values of $\Phi_{ij...n}$ and $f(x_i,x_j,....,x_n)$

which depend on n variables. The remaining steps are likewise adapted to the existence of n variables. All derivatives, of course, will be partial derivatives. The construction of the final image from the temporary image - step (8) - can likewise be carried out with obvious generalizations in the case of images defined in a space having any number of dimensions.

II - A surface can be considered as a family of lines, which are obtained by the intersection of the surface with a family of planes, e.g. vertical planes the orientation of which is taken as that of the x-axis, identified by a parameter, e.g. their y coordinate. A family of curves in a plane, depending on one parameter, as may result from the representation of any number of phenomena, is obviously equivalent to that of a surface and may be treated as such, or vice versa.

III - A particular case of an object which is a surface is, e.g., a terrain, wherein the surface is defined by the elevation as a function of two plane (cartesian or polar) coordinates.

IV - A building can be represented in the same way, if it is very simple. If its shape is complex, however, it must be broken up into a number of component parts. However, if it is desired to represent it as it is seen from the outside, say by an observer which can place himself at any vantage point within a certain distance from the building, the oberver's position can be identified by three coodinates, x, y and z (or polar coordinates), or by two, if it assumed that the observer's eye is at a given level. From each position of the observer point it is possible, if the configuration of the terrain is

known, to determine the distance D on each line of sight from the observer's eye to the building surface, and this will determine how the building is seen. Each line of sight can be identified by two coordinates: e.g. its inclination (the angle thereof with the vertical in a vertical plane which contains it), and its azimuth (the angle of said vertical plane with a reference vertical plane, e.g. one that contains the geographic or magnetic north). The way in which the building appears to the observer, is therefor defined by a function D of five variables, viz. by a surface in a six-dimensional space.

V - A family of curves in a plane, depending on more than one parameter, is obviously equivalent to a surface in a space having more than three dimensions.

VI- If in example IV above the coordinates of the observer are known as a function of a single variable, say, when he approaches the building along a given line, in which case the variable is the distance covered from a starting point, or in motion, as in a vehicle, along a given line, in which case the variable is time. In this case the variables of the surface become three (e.g. distance or time and inclination and azimuth) and the space is only four-dimensional, but the four-dimensional surface is subject to the constraint represented by the definition of the observer's motion. In general, in many cases, the degree of the space in which the surface is defined may be reduced by the introduction of suitable constraints.

VII - The final image of a colour picture is another colour picture, that is not identical, but sufficiently similar to the object picture. The object picture can be scanned by known apparatus (scanners), by means of white light, and for each point the intensity of the three basic colours (magenta, cyan and yellow) may be measured and registered. The object is thus reduced to three partial or component objects, each consisting of the distribution of one basic colour over the picture and having a physical reality, as it r equivalent to the colour picture that would be contained by exposing the original through three filters, having colours complementary to the three basic colours, or, in practice, to an array of digital data representing such one-coloured picture. Each of said partial objects can be subjected to the process of the invention, to produce a reduced or compressed array of data, constituting a partial image, and the partial images can be transformed into a combined final image approximating the original object, by processes known to those skilled in the art. If the partial images must be stored and/or transmitted, the process of the invention will facilitate doing this and render it more economical. In the same way a dynamic coloured picture, such as a movie or a TV broadcast, can be reduced to a final dynamic image.

A particular advantage and a preferred aspect of the invention consists in the possibility of processing the compressed intermediate image obtained as set forth hereinbefore and producing from it a processed final image, which does not represent the object but represents what would have been the result of processing the object. The processed intermediate image can be stored and transmitted with the already mentioned savings and

advantages inherent in the reduction of the number of data, but said reduction is even more advantageous in the processing, for it is obviously more convenient to process a reduced instead of a larger amount of data.

Said processing in a compressed form, as it may be called, is made possible by the following property:

Let F be an operator which is analytic in nature, viz. can be defined by mathematical relationships. Let O be an object of any nature, but which can be represented by Taylor polynomials p_i. Then by applying operator F to the p_i's, one obtains polynomials which represent the object that would be obtained by applying the operator F to the object O. If one uses the symbol $\approx$ to indicate that an array of polynomials represents an object, one can write:

$$\text{if } p_i \approx O, \text{ then } F(p_i) \approx F(O).$$

Elementary examples of analytic operators are algebraic operations, rotations of geometrical figures, changes of coordinates in general, etc. These operators are represented by mathematical operations. If F(O) is to be constructed, such operations must be carried out on all the data, e.g. digital data, which define the object. But if a compressed image has been obtained as set forth above, and an array of Taylor polynomial coefficients has been obtained, which are in a much smaller number than the said digital data, said mathematical operations can be carried out on said coefficients, and a processed intermediate image will be obtained, which represents F(O) and from which F(O) can be constructed as set forth in step (9) above.

The following examples illustrate a number of embodiments of the invention.

Example 1

An object line f in the plane (x, y) is given by an array

$A = (y_0, y_1, ..., y_{100})$, where $y_i = f(x_i)$, $x_i = i/100$, $i = 0, 1, ... , 100$. In this specific example the array (array 1) is the following:

0.1152 0.1155 0.1191 0.1131 0.1174 0.1133 0.1108 0.1149 0.1105

0.1182 0.1167 0.1206 0.1238 0.1196 0.1264 0.1282 0.1313 0.1315

0.1299 0.1330 0.1366 0.1409 0.1402 0.1462 0.1569 0.1608 0.1631

0.1604 0.1693 0.1779 0.1797 0.1826 0.1826 0.1888 0.1963 0.2011

0.2034 0.2084 0.2170 0.2244 0.2265 0.2327 0.2429 0.2468 0.2472

0.2523 0.2661 0.2673 0.2702 0.2796 0.2811 0.2845 0.2949 0.3022

0.3078 0.3049 0.3121 0.3157 0.3256 0.3270 0.3346 0.3413 0.3405

0.3428 0.3503 0.3515 0.3530 0.3571 0.3675 0.3616 0.4648 0.4665

0.4659 0.4607 0.4600 0.4536 0.4473 0.4441 0.4427 0.4330 0.4329

0.4268 0.4243 0.4185 0.4135 0.4107 0.3961 0.3925 0.3877 0.3774

0.3698 0.3671 0.3583 0.3449 0.3397 0.3338 0.3271 0.3091 0.3031

0.2929

An object line itself is shown in Fig. 3a. The required accuracy of representing this line is 0.035. The compressed image of this line is produced as follows.

Firstly it is subdivided into three segments lying over the segments [0.0, 0.6], [0.6, 0.8], [0.8, 1.0] in the x-axis. The following model is chosen on the segments [0.0, 0.6] and [0.8, 1.0]:

$y = Q(x) = c_1 \sin(\omega_1 x + \phi_1) + c_2 \cos(\omega_2 x + \phi_2) + c_3 x^2 + c_4 x + c_5$ with c_1, c_2, ω_1, ω_2, ϕ_1, ϕ_2, c_3, c_4, c_5 - the parameters.

On the segment [0.6, 0.8] the following model is chosen:

$y = Q(x) + H x_0$, a, b, c, d (x), where $Q(x)$ is as above, and the normal form H is defined by $H(x) = a(x - x_0) + b$, if $x \geq x_0$ or $H(x) = c(x - x_0) + d$, if x is less than x_0. Said normal form is illustrated in Fig. 3d. Approximation on each segment is carried out by minimization, with respect to the corresponding parameters, of the quadratic error:

$\Sigma (y_i - Q(x_i))^2$ $(\Sigma (y_i - Q(x_i) - H(x_i))^2$ on [0.6, 0.8]).

The values of the parameters found are given in the following array 2.

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Q(x) = 2.0  + 0.1*x - 0.2*x*x - 0.15*cos( -0.4+4*x) -
       0.2*sin( -0.3 + 0.5*x )

H(x) =  1.0/7.0 *  (x-0.7) + 0.1  ,   x < 0.7
H(x) = -1.0/3.0 *  (x-0.7) + 0.2  ,   x >= 0.7
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The corresponding model curve is shown in Fig. 3b.

The error of the approximation of the object line by the model found turns out to be 0.005. Respectively, on the step 2, ε is chosen to be 0.03. k is chosen to be 2 on each segment.

M, equal to the maximal absolute value of the third derivative of the smooth component in the above model, as computed by the standard subroutine, 8. The maximal possible pitch h of the grid to be constructed, is defined by $(1/6) M (h/2)^3 = \varepsilon$, or $h \approx 0.24$. In order to provide a uniform grid, a smaller value $h = 0.2$ is chosen on each segment. The corresponding grid points are the following: 0.1, 0.3, 0.5 on [0.0, 0.6], the only grid point 0.7 on [0.6, 0.8] and the only grid point 0.9 on [0.8, 1.0]. Taylor polynomials at these points, as computed by the standard "MATHEMATICA" subroutine, are given in the following array 3.

Zi	a0	a1	a2
0.1	0.121582031250	0.105957031250	0.993652343750
0.3	0.180175781250	0.454345703125	0.632080078125
0.5	0.285644531250	0.542724609375	-0.236083984375
0.9	0.381103515625	-0.727050781250	-1.394042968750
0.7	0.272460937500	0.125244140625	-1.083496093750

a= 0.142822265625   b= 0.099853515625

c= -0.333251953125   d= 0.199951171875

Now the coefficients of order 0 are rounded off up to 3 digits, the coefficients of order 1 are rounded off up to 2 digits and the coefficients of order 2 up to 1 digit. The parameters of the normal form H are rounded off up to three digits. These data, listed in the following array 4 represent the intermediate compressed image.

zi	a0	a1	a2
0.1	0.121	0.10	0.9
0.3	0.180	0.45	0.6
0.5	0.285	0.54	-0.2
0.9	0.381	-0.72	-1.3
0.7	0.272	0.12	-1.0

a= 0.142   b= 0.100

c= -0.333   d= 0.200

The compression ratio is 4*100 digits/ 37 digits ≈ 10.8.

The final image is obtained by computing the values of the Taylor polynomials (and the normal form H on [0.6, 0.8]) at the initial points x, i = 0, ..., 100. Each polynomial is used for x, belonging to the corresponding cell of the grid z_i. The result is shown in the following array 5.

0.1196 0.1190 0.1185 0.1183 0.1182 0.1183 0.1186 0.1190 0.1197

0.1205 0.1215 0.1227 0.1240 0.1256 0.1273 0.1292 0.1313 0.1335

0.1360 0.1386 0.1426 0.1460 0.1496 0.1532 0.1570 0.1609 0.1649

0.1691 0.1733 0.1777 0.1822 0.1868 0.1916 0.1964 0.2014 0.2065

0.2117 0.2171 0.2225 0.2281 0.2318 0.2376 0.2433 0.2490 0.2546

0.2602 0.2658 0.2713 0.2768 0.2822 0.2876 0.2930 0.2983 0.3036

0.3088 0.3140 0.3192 0.3243 0.3294 0.3344 0.3380 0.3424 0.3466

0.3506 0.3545 0.3581 0.3615 0.3648 0.3678 0.3706 0.4709 0.4685

0.4660 0.4633 0.4603 0.4572 0.4539 0.4503 0.4466 0.4427 0.4376

0.4328 0.4276 0.4223 0.4166 0.4107 0.4046 0.3981 0.3915 0.3845

0.3773 0.3699 0.3621 0.3542 0.3459 0.3374 0.3287 0.3196 0.3104

0.3008

The corresponding final curve is shown in Fig. 3c. The maximal error in representing the object curve by the final one is 0.033.

Example 2

The object (black and white, continuous tone) picture is the standard test picture, called "Lena" (see Fig. 4a). It is represented by a 512 x 512 array, each pixel containing 8 bits, representing one of the gray levels between 0 and 255. The file representing this picture is available in test collections in the field of imaging . A part of this array, representing the piece S, marked on Fig. 4a, is the following.

```
 97  97  97  97  97  97 100  98  96  94  92  90  89  86  84  84  86
 89  93  96 101 109 120 133 152 171 188 202 213 222 214 200 185 167
148 127 138 136 133 130 128 125 115 114 113 111 110 109

 97  97  97  97  97  97  99  97  95  93  91  89  89  85  83  83  85
 89  92  95 100 108 119 132 150 169 186 200 211 220 217 203 187 169
149 127 138 135 132 130 127 124 115 114 113 111 110 109

 97  97  97  97  97  97  98  96  94  92  90  88  88  84  83  83  84
 88  92  94 100 108 118 132 148 167 184 198 209 218 221 206 189 171
150 128 137 134 132 129 126 124 115 114 113 111 110 109

 97  97  97  97  97  97  98  96  94  92  90  88  87  84  82  82  84
 87  91  94  99 107 118 131 146 165 182 196 207 216 224 208 191 172
152 129 136 134 131 128 126 123 115 114 113 111 110 109

 97  97  97  97  97  97  97  95  93  91  89  87  87  83  81  81  83
 87  90  93  98 106 117 130 144 163 180 194 205 214 227 211 193 174
153 130 136 133 130 128 125 122 115 114 113 111 110 109

 97  97  97  97  97  97  96  94  92  90  88  86  86  82  81  81  82
 86  90  92  98 106 116 130 142 161 178 192 203 212 230 214 196 176
154 131 135 132 130 127 124 122 115 114 113 111 110 109

 99  96  94  93  93  94  93  94  93  92  90  87  85  85  85  85  85
 85  86  90  96 104 114 125 136 145 158 175 195 219 220 222 213 193
161 117 134 132 129 127 125 122 121 118 115 111 108 105

 95  93  91  91  91  92  95  95  95  93  91  88  84  84  84  84  84
 84  87  91  97 104 113 124 134 144 157 174 194 218 220 223 215 195
163 120 133 131 129 127 125 123 121 118 115 111 108 105

 92  90  89  88  89  91  96  96  96  95  92  89  84  84  84  84  84
 84  88  92  97 104 112 123 133 143 156 172 193 216 220 224 216 196
165 122 131 129 128 126 125 123 121 118 115 111 108 105

 88  86  86  86  87  89  97  98  97  96  94  91  84  84  84  84  84
 84  89  92  97 103 112 122 132 141 154 171 191 215 220 224 217 198
167 125 128 127 125 124 123 122 121 118 115 111 108 105

 84  83  83  83  85  87  99  99  99  97  95  92  86  86  86  86  86
 86  91  93  97 103 111 121 130 140 153 170 190 214 221 225 218 199
169 127 124 123 122 122 121 120 121 118 115 111 108 105

 80  80  80  81  83  86 100 100 100  99  96  93  88  88  88  88  88
 88  92  94  97 103 110 119 129 139 152 168 189 212 221 226 219 201
171 130 119 119 118 118 118 118 121 118 115 111 108 105
```

```
 65  71  78  85  91  98 100 100 100 101 101 102 102 100  97  94  89
 83  91  91  93  97 105 115 128 143 157 172 187 201 223 229 223 205
174 130 118 118 118 118 118 118 118 114 111 107 103  99


 65  72  79  85  92  99 104 104 104 104 104 104 107 105 103  99  95
 90  89  88  90  95 102 112 128 143 158 172 187 202 219 226 220 202
172 129 118 118 118 118 118 118 115 112 109 105 102  99


 66  73  79  86  93  99 106 105 105 104 104 104 108 107 105 102  98
 94  86  85  87  92  99 109 127 141 156 171 185 200 214 221 216 198
168 126 118 118 118 118 118 118 113 110 108 105 102  99


 67  73  80  87  93 100 105 104 103 102 101 100 107 106 105 102  99
 95  83  83  85  89  97 107 123 138 153 167 182 197 207 214 210 193
163 121 118 118 118 118 118 118 112 110 107 105 102 100


 67  74  81  87  94 101 101 100  99  97  96  95 103 103 102 100  97
 93  81  80  82  87  94 104 118 133 148 162 177 192 198 206 202 185
156 114 118 118 118 118 118 118 112 110 108 106 104 102


 68  75  81  88  95 101  95  93  92  90  88  86  97  97  96  95  92
 89  78  77  79  84  91 101 111 126 141 155 170 185 187 196 192 176
147 106 118 118 118 118 118 118 113 111 110 108 106 105


 79  83  88  93  97 102  94  69  48  30  15   4  40  51  59  65  68
 68  80  80  82  85  88  93  94 110 128 146 164 184 192 176 160 146
132 120 116 115 115 115 115 114 107 106 104 102  98  94


 80  83  86  90  93  96  89  68  50  36  26  19  44  53  59  62  63
 61  71  72  74  76  80  84  86 102 119 137 156 176 185 171 157 145
133 122 117 116 115 115 114 113 109 108 106 104 100  96


 81  83  85  87  89  91  86  69  56  46  39  36  53  59  63  64  63
 58  66  67  69  72  75  80  81  97 114 132 151 171 179 166 155 144
134 125 118 117 116 114 113 112 110 109 107 104 101  96


 84  85  85  86  87  87  86  73  64  58  55  56  66  70  71  70  67
 60  65  66  68  70  74  78  80  96 113 131 150 170 173 162 152 143
134 127 119 117 116 114 113 111 110 109 107 104 101  96


 88  87  86  86  85  84  89  80  74  72  74  79  83  85  84  81  75
 66  67  68  70  72  76  80  82  98 115 133 152 172 166 157 149 141
135 129 120 118 116 114 112 110 109 108 106 104 100  96


 92  90  88  86  84  82  94  89  88  90  95 104 105 105 102  96  88
 77  73  74  76  78  82  86  88 104 121 139 158 178 160 152 146 140
136 132 121 119 116 114 111 109 107 106 104 102  98  94
```

```
107 104 101  98  94  91  86  84  83  83  84  86  91  97  98  93  83
 68  62  71  82  96 113 132 158 162 166 168 170 171 154 148 141 134
128 121 124 122 119 116 114 111 105 104 102 101 100  98


113 110 107 105 102  99  91  89  88  88  89  91  94 103 106 104  97
 84  87  96 107 121 138 157 169 172 174 176 176 176 154 147 140 134
127 120 124 121 118 116 113 110 104 103 102 100  99  98


117 114 112 110 108 106  95  94  93  93  94  95  99 111 117 117 113
103 105 114 125 139 156 175 178 179 180 180 179 178 153 146 140 133
126 120 123 120 118 115 112 110 104 102 101 100  98  97


119 117 115 114 112 110 100  98  97  97  98 100 106 120 129 132 130
123 116 124 136 150 166 186 184 184 184 182 180 177 152 146 139 132
126 119 122 120 117 114 112 109 103 102 100  99  98  96


119 118 117 115 114 113 105 103 102 102 103 105 114 131 142 148 149
144 119 128 139 153 170 189 187 186 184 182 178 174 152 145 138 132
125 118 122 119 116 114 111 108 102 101 100  98  97  96


118 117 116 115 114 114 109 108 107 107 108 109 125 144 158 167 170
168 116 124 136 150 166 186 188 185 182 178 173 168 151 144 138 131
124 118 121 118 116 113 110 108 102 100  99  98  96  95


118 118 118 118 118 118 111 110 111 114 118 125 131 156 173 183 184
177 152 145 145 153 169 194 185 184 182 180 179 177 154 145 136 129
123 117 112 114 114 112 108 102  96 108 111 107  95  75


118 118 118 118 118 118 114 113 114 117 121 127 136 161 178 186 187
180 157 148 147 155 170 193 188 186 184 182 180 178 153 144 136 128
122 116 112 114 114 112 108 102  98 107 108 100  85  62


118 118 118 118 118 118 117 116 117 119 124 130 141 166 182 190 190
183 161 151 150 156 171 193 190 188 185 183 180 178 152 143 135 128
121 116 112 114 114 112 108 102 101 106 104  93  75  49


118 118 118 118 118 118 119 118 119 122 126 133 147 170 186 194 194
185 165 155 152 158 171 193 191 188 185 182 180 177 152 143 134 127
121 115 112 114 114 112 108 102 103 106 100  87  65  35


118 118 118 118 118 118 122 121 122 125 129 135 152 175 190 198 197
188 169 158 155 159 172 193 191 188 185 181 178 175 151 142 134 126
120 114 112 114 114 112 108 102 106 105  96  80  55  22


118 118 118 118 118 118 125 124 125 127 132 138 157 180 195 201 200
191 174 161 157 161 173 192 191 187 183 179 176 172 150 141 133 126
119 114 112 114 114 112 108 102 108 104  93  73  45   9
```

```
114 117 119 122 125 127 120 123 126 130 136 142 160 176 188 195 198
196 158 152 151 154 162 174 198 191 184 178 173 169 160 140 125 116
112 113 112 111 110 109 108 107 121  90  64  44  29  19

112 115 118 120 123 126 124 126 128 132 136 141 168 184 195 202 204
202 175 168 166 168 174 185 200 194 189 185 182 180 161 140 125 115
110 111 111 110 109 109 108 107 111  82  58  39  26  18

111 114 117 119 122 125 128 128 130 132 136 140 168 183 194 200 202
199 186 178 174 174 180 189 193 189 186 184 183 182 160 139 123 113
108 108 109 109 108 108 108 108 102  74  52  35  23  17

111 114 117 119 122 125 130 130 130 132 134 138 158 173 183 189 190
187 190 181 176 175 179 187 179 177 175 175 175 177 158 137 121 110
104 104 107 107 108 108 108 108  92  66  46  30  21  16

112 115 118 120 123 126 131 130 130 130 132 134 140 154 164 169 170
166 189 178 172 170 172 179 156 156 156 158 160 163 156 134 117 106
100  99 105 106 107 107 108 109  82  58  39  26  18  15

114 117 119 122 125 127 132 130 128 128 129 130 112 126 136 140 141
137 181 169 161 158 159 164 126 127 130 133 137 142 153 130 113 101
 95  94 104 105 106 107 108 109  72  50  33  22  15  14

117 115 113 112 113 114 109 105 102 101 100 100  97 102 105 108 110
111 134 134 131 127 121 113 110 113 116 119 122 125 111 100  93  91
 93 100  91 110 120 121 114  97  29  27  25  24  22  20

100  98  96  96  96  97  95  92  90  88  88  88  87  91  95  97  99
100 117 116 113 108 102  94  81  85  89  93  97 101  91  82  78  78
 82  91 100 114 120 116 104  83  25  24  23  21  20  19

 86  84  82  82  82  83  82  80  78  77  77  78  77  82  85  88  90
 91 103 102  98  93  86  78  61  66  71  76  81  86  78  72  69  72
 78  89 107 117 118 110  94  68  23  22  21  20  19  18

 75  73  71  70  71  72  71  68  67  67  67  68  70  75  78  81  83
 83  93  91  87  82  74  65  50  56  62  68  73  79  73  68  68  73
 82  95 113 119 116 103  82  52  21  20  20  20  19  19

 66  64  62  62  62  63  60  58  57  57  58  60  65  69  73  75  77
 78  86  84  80  74  66  56  48  55  62  68  75  82  74  72  74  81
 92 108 119 120 112  96  70  36  20  20  20  20  20  20

 60  58  56  56  56  57  50  48  48  48  50  52  61  65  69  71  73
 74  83  80  76  69  61  51  55  63  70  78  85  93  83  83  87  96
110 127 124 120 108  87  57  18  20  20  21  21  22  22
```

The compressed image is produced as follows: first, the picture is subdivided into square segments, c ontaining 6 x 6 = 36 pixels each one. See Fig. 4a and the array S above, where one of such segments is marked.

The step 1 consists in approximating the picture on each segment by the model, which is chosen to be the quadratic polynomial

$$z = a_0 + a_1 x + a_2 y + a_{11} x^2 + 2a_{12} xy + a_{22} y^2$$

where z represents the gray level, and x and y are the coordinates on t picture plane centered at the center of the corresponding segment.

The values of the coefficient "a" are found by the standard subroutine, minimizing the quadratic error of the approximation of the gray level on each segment by the model chosen.

The array of 8 x 8 = 64 polynomials, obtained on the segments, covering the piece S of the picture, is given in the following array 7.

0.37500000	-0.0058594	-0.0055246	0.001883	-0.007146	0.010986
0.35937500	-0.0104353	-0.0276228	0.007010	0.013632	0.009208
0.31640625	-0.0081473	0.0078125	0.010149	-0.001263	0.033064
0.39843750	-0.0127790	0.0998326	0.003348	0.014263	0.062360
0.74218750	-0.0247767	0.1643973	-0.015172	0.000459	-0.049700
0.70703125	0.0304130	-0.2262835	-0.009312	-0.027809	-0.038191
0.50781250	-0.0092076	-0.0348772	-0.015067	-0.003071	0.007847
0.43359375	0.0021763	-0.0233817	0.007010	0.004908	-0.004918
0.33593750	-0.0344308	-0.0031808	0.003348	0.027637	0.026263
0.37109375	0.0164063	-0.0194196	-0.005022	0.008954	-0.022600
0.32421875	0.0092634	-0.0077009	0.016532	-0.015383	-0.003871
0.38671875	-0.0045201	0.0807478	0.006069	-0.026834	0.035575
0.63671875	-0.0199777	0.1973214	0.009626	-0.011422	0.066127
0.81250000	0.0224331	-0.2329241	0.003558	0.016875	-0.205811
0.48828125	-0.0261161	-0.0174107	-0.025321	0.024337	0.006069
0.43750000	0.0035156	-0.0440290	-0.002511	-0.010188	-0.007533
0.32031250	0.0101563	0.0811384	0.006278	0.006256	0.002511
0.40234375	-0.0292969	-0.0090960	-0.058594	-0.026260	0.015172
0.40234375	-0.0013951	-0.0376674	-0.050642	0.022643	-0.022391
0.33984375	-0.0376116	0.0617188	0.004290	-0.008265	0.046980
0.63281250	-0.0425223	0.1786830	-0.036516	0.004305	0.006173
0.80468750	-0.0712053	-0.2079241	-0.040388	0.029043	-0.234375
0.45703125	-0.0006696	0.0003348	-0.000314	-0.009184	-0.011300
0.41015625	0.0016741	-0.0319196	0.020299	0.019056	-0.000732
0.33203125	-0.0089286	0.0193080	0.020194	-0.052519	0.014544
0.21093750	0.1237165	-0.0967076	0.060059	0.156036	0.077009
0.25781250	0.0914063	0.0062500	0.087995	-0.093673	-0.055455
0.26562500	-0.0221540	0.0343192	0.074916	-0.002899	0.018101
0.47265625	-0.0229911	0.2156250	0.078055	0.008839	0.015904
0.57812500	-0.0305245	-0.1201451	-0.003557	0.072006	0.021240
0.44531250	0.0000558	-0.0220424	0.014230	-0.028096	0.001674
0.41015625	-0.0073103	-0.0318638	-0.019148	0.008409	-0.016950
0.43750000	0.0410714	-0.0239955	-0.033378	0.020663	-0.000732
0.36718750	0.0566406	0.0056362	0.014962	-0.005309	0.016218
0.48046875	0.1614397	0.0278460	0.038295	0.096687	-0.109236
0.53906250	0.1313058	0.1659040	-0.128697	0.012025	0.054618
0.71093750	0.0371094	-0.0090960	-0.059326	-0.061617	-0.031076
0.53125000	-0.0079799	-0.0807478	0.012347	-0.010418	0.015486
0.44921875	-0.0116630	-0.0317522	-0.008789	0.007950	0.000000
0.38671875	-0.0111607	-0.0160714	-0.011300	0.000402	0.000000
0.45703125	0.0071429	-0.0061384	0.010568	0.007691	-0.002302
0.46093750	0.0342634	0.0351562	0.001988	0.005568	0.035261
0.73828125	0.0527344	0.0965960	-0.012660	-0.018109	-0.150774
0.59765625	0.0304687	0.0774553	-0.012556	-0.041212	0.146589
0.71375000	0.0060268	-0.0338170	-0.019880	-0.024452	-0.011405
0.51171875	-0.0079799	-0.0932478	-0.001569	-0.002612	0.015695
0.43750000	-0.0044085	-0.0244978	0.009312	-0.004334	-0.033064
0.37500000	-0.0677456	-0.1410714	0.003557	-0.121397	-0.143032
0.45703125	-0.0024554	0.0328125	0.024484	0.014522	-0.005650
0.51171875	-0.0028460	0.0307478	-0.024170	-0.036993	0.026681
0.75390625	-0.1251674	0.0778460	-0.168248	-0.019142	-0.088518
0.68359375	0.0162947	0.0053571	-0.122001	-0.059694	0.078265
0.70703125	-0.1199218	-0.0175781	-0.155797	0.070226	0.024379
0.44921875	-0.0365513	-0.1284040	-0.030866	-0.027522	0.101597
0.41796875	-0.0125000	0.0001116	0.000628	0.019687	-0.008161
0.15234375	-0.0695313	-0.1944197	0.006906	0.062679	0.108608
0.29296875	-0.1353237	-0.0140067	0.060582	0.000373	0.015695
0.27734375	-0.1326451	-0.0152344	0.019357	0.018683	0.025635
0.32031250	-0.0872768	0.0389509	0.044155	-0.000804	-0.024902
0.34765625	-0.1336496	-0.0653460	0.070103	-0.026145	-0.042585
0.26171875	-0.1049665	0.0693639	0.164062	0.038772	-0.000419
0.26562500	0.0065848	0.0437500	0.130371	0.086556	0.090193
0.43750000	-0.0600446	-0.1228795	-0.031076	-0.157902	-0.156948
0.07421875	-0.0148995	-0.0127790	0.015904	0.030048	0.013079

(The coefficients are given after rescaling the x and y variables to the square [-1, 1] [-1, 1], and the gray level z to [0, 1]).

Step 2 The required accuracy ε is chosen to be 5 gray levels, k is fixed to be 2, and the grid on each segment is chosen to contain the only point - the center of this segment. Thus the Taylor polynomials computed on this step are identical to the approximating polynomials found on the step 1.

The 6 digits accuracy with which the coefficients of these polynomials are given in the array P above is excessive, and the coefficients are rounded off up to 8 bits in degree 0, up to 7 bits in degree 1 and up to 6 bits in degree 2.

The corresponding binary array is the intermediate compressed image. It is approximately represented by the following digital array P' (corresponding to the same piece S of the picture, as the above array P).

0.37500000	0.0000000	0.0000000	0.000000	0.000000	0.000000
0.35937500	-0.0078125	-0.0234375	0.000000	0.000000	0.000000
0.31640625	-0.0078125	0.0000000	0.000000	0.000000	0.031250
0.39843750	-0.0078125	0.0937500	0.000000	0.000000	0.046875
0.74218750	-0.0234375	0.1640625	0.000000	0.000000	-0.046875
0.70703125	0.0234375	-0.2187500	0.000000	-0.015625	-0.031250
0.50781250	-0.0078125	-0.0312500	0.000000	0.000000	0.000000
0.43359375	0.0000000	-0.0156250	0.000000	0.000000	0.000000
0.33593750	-0.0312500	0.0000000	0.000000	0.015625	0.015625
0.37109375	0.0156250	-0.0156250	0.000000	0.000000	-0.015625
0.32421875	0.0078125	0.0000000	0.015625	0.000000	0.000000
0.38671875	0.0000000	0.0781250	0.000000	-0.015625	0.031250
0.63671875	-0.0156250	0.1953125	0.000000	0.000000	0.062500
0.81250000	0.0156250	-0.2265625	0.000000	0.015625	-0.203125
0.49218750	-0.0234375	-0.0156250	-0.015625	0.015625	0.000000
0.43750000	0.0000000	-0.0390625	0.000000	0.000000	0.000000
0.32031250	0.0078125	0.0781250	0.000000	0.000000	0.000000
0.40234375	-0.0234375	-0.0078125	-0.046875	-0.015625	0.000000
0.40234375	0.0000000	-0.0312500	-0.046875	0.015625	-0.015625
0.33984375	-0.0312500	0.0546875	0.000000	0.000000	0.046875
0.63281250	-0.0390625	0.1718750	-0.031250	0.000000	0.000000
0.80468750	-0.0703125	-0.2031250	-0.031250	0.015625	-0.218750
0.45703125	0.0000000	0.0000000	0.000000	0.000000	0.000000
0.41015625	0.0000000	-0.0312500	0.015625	0.015625	0.000000
0.33203125	-0.0078125	0.0156250	0.015625	-0.046875	0.000000
0.21093750	0.1171875	-0.0937500	0.046875	0.140625	0.062500
0.25781250	0.0859375	0.0000000	0.078125	-0.078125	-0.046875
0.26562500	-0.0156250	0.0312500	0.062500	0.000000	0.015625
0.47656250	-0.0156250	0.2109375	0.062500	0.000000	0.015625
0.57812500	-0.0234375	-0.1171875	0.000000	0.062500	0.015625
0.44531250	0.0000000	-0.0156250	0.000000	-0.015625	0.000000
0.41015625	0.0000000	-0.0312500	-0.015625	0.000000	-0.015625
0.43750000	0.0390625	-0.0234375	-0.031250	0.015625	0.000000
0.36718750	0.0546875	0.0000000	0.000000	0.000000	0.015625
0.48437500	0.1562500	0.0234375	0.031250	0.093750	-0.093750
0.53906250	0.1250000	0.1640625	-0.125000	0.000000	0.046875
0.71093750	0.0312500	-0.0078125	-0.046875	-0.046875	-0.015625
0.53125000	-0.0078125	-0.0781250	0.000000	0.000000	0.000000
0.44921875	-0.0078125	-0.0312500	0.000000	0.000000	0.000000
0.38671875	-0.0078125	-0.0156250	0.000000	0.000000	0.000000
0.45703125	0.0000000	0.0000000	0.000000	0.000000	0.000000
0.46093750	0.0312500	0.0312500	0.000000	0.000000	0.031250
0.73828125	0.0468750	0.0937500	0.000000	-0.015625	-0.140625
0.59765625	0.0234375	0.0703125	0.000000	-0.031250	0.140625
0.71875000	0.0000000	-0.0312500	-0.015625	-0.015625	0.000000
0.51171875	-0.0078125	-0.0859375	0.000000	0.000000	0.015625
0.43750000	0.0000000	-0.0234375	0.000000	0.000000	-0.031250
0.37500000	-0.0625000	-0.1406250	0.000000	-0.109375	-0.140625
0.45703125	0.0000000	0.0312500	0.015625	0.000000	0.000000
0.51171875	0.0000000	0.0234375	-0.015625	-0.031250	0.015625
0.75390625	-0.1250000	0.0703125	-0.156250	-0.015625	-0.078125
0.68359375	0.0156250	0.0000000	-0.109375	-0.046875	0.078125
0.70703125	-0.1171875	-0.0156250	-0.140625	0.062500	0.015625
0.44921875	-0.0312500	-0.1250000	-0.015625	-0.015625	0.093750
0.41796875	-0.0078125	0.0000000	0.000000	0.015625	0.000000
0.15234375	-0.0625000	-0.1875000	0.000000	0.062500	0.093750
0.29296875	-0.1328125	-0.0078125	0.046875	0.000000	0.015625
0.27734375	-0.1250000	-0.0078125	0.015625	0.015625	0.015625
0.32031250	-0.0859375	0.0312500	0.031250	0.000000	-0.015625
0.34765625	-0.1328125	-0.0625000	0.062500	-0.015625	-0.031250
0.25171875	-0.1015625	0.0625000	0.156250	0.031250	0.000000
0.26562500	0.0000000	0.0390625	0.125000	0.078125	0.078125
0.43750000	-0.0546875	-0.1171875	-0.015625	-0.156250	-0.156250
0.07421875	-0.0078125	-0.0078125	0.015625	0.015625	0.000000

The compression ratio is 512*512*8 bits/86*86*(8 + 2*7 + 3*6) bits ≈ 6.7.

The final image is obtained by computing the values of the Taylor polynomials, representing the intermediate image, at each pixel of the corresponding segment. The part S' of the obtained array, representing the final image (and corresponding to the piece S of the initial picture), is the following array 9.

```
137 137 137 136 138 129 138 134 140 136 135 134 130 139 135 129 134
131 138 139 127 132 129 134 131 133 130 125 130 129 127 131 127 134
128 135 133 144 135 142 143 142 143 150 148 154 153 149


149 149 143 154 145 142 135 133 118 120 127 110  98  82  86  74  63
 65  63  61  55  67  72  66  73  69  73  74  73  73  70  78  80  76
 75  73  72  80  80  78  77  75  75  76  78  78  75  78


 78  79  78  78  73  75  73  75  79  79  89  77  81  86  90  86  78
 89  89  92  95  92  90  92  91 101  97  92  89  94 104 100 101  96
104  98  96  98 100  95 106 108 102 103 100 100  96  99


104  91  98  95 105 101  98 101  97 102  97  97 105 104 105 103  95
102  97 105 102 102 103 103  99 103  96 106 102  99 102 101 106 100
104 103 101 102 100 100 100 104 108 103 104  99 104  99


105 106 101 105 105 105 101 109 106 104 102 105 104 105 102 104 112
102  99 110 104 101 100 102 102  98 100 103 103  99  98 100 101  97
107 105 104 106 101 100 100 101  99 102  97  98  99 103


105 103 101 103 101 105 104 109 102  99 102 103  96  96 103 108 109
114 104 102 102  97  99  97 100 102  98 101 102 102  99  98  99  97
100 102  93 100 105 105  99  97  96 102  96 101 104  94


 95 103  99  95  99  95 102  93  97  99  92  96  95 102  95 101  90
 94  93  89  86  93  91  90  89  86  83  85  87  74  72  73  71  82
 83  88  90 102 106 111 110 116 118 125 128 126 127 136


136 136 141 135 142 141 131 130 121 122 129 122 130 125 123 131 125
131 129 126 131 130 128 128 135 133 129 132 129 127 124 133 127 126
128 127 129 126 127 127 126 124 124 136 129 129 128 127


129 134 129 136 135 134 129 130 131 136 136 136 132 132 132 127 131
134 134 161 181 189 198 201 206 207 206 210 212 213 211 213 215 210
204 197 183 156 131  96  77  77  71  74  76  83  73  85


 87  89  91  88  90  90  94  93 101  93  88  88  92  96  92  87  94
 93  98 101  92  90  94  90  89  92  94  88  93  88  91  90  95  93
100  86  93  97  94  97  96  97  93  95  94  94  87  86


100  92  97  96  95 101  95  97  93  89 101  95  97  97  88  94  82
 91  84  88  89  84  91  93 107 139 144 147 149 148 130  99 137 137
137 136 138 129 138 134 140 136 135 134 130 139 135 129


134 131 138 139 127 132 129 134 131 133 130 125 130 129 127 131 127
134 128 135 133 144 135 142 143 142 143 150 148 154 153 149 149 149
143 154 145 142 135 133 118 120 127 110  98  82  86  74
```

```
63  65  63  61  55  67  72  66  73  69  73  74  73  73  70  78  80
76  75  73  72  80  80  78  77  75  75  76  78  78  75  78  78  79
78  78  73  75  73  75  79  79  89  77  81  86  90  86


78  89  89  92  95  92  90  92  91 101  97  92  89  94 104 100 101
96 104  98  96  98 100  95 106 108 102 103 100 100  96  99 104  91
98  95 105 101  98 101  97 102  97  97 105 104 105 103


95 102  97 105 102 102 103 103  99 103  96 106 102  99 102 101 106
100 104 103 101 102 100 100 100 104 108 103 104  99 104  99 105 106
101 105 105 105 101 109 106 104 102 105 104 105 102 104


112 102  99 110 104 101 100 102 102  98 100 103 103  99  98 100 101
97 107 105 104 106 101 100 100 101  99 102  97  98  99 103 105 103
101 103 101 105 104 109 102  99 102 103  96  96 103 108


109 114 104 102 102  97  99  97 100 102  98 101 102 102  99  98  99
97 100 102  93 100 105 105  99  97  96 102  96 101 104  94  95 103
99  95  99  95 102  93  97  99  92  96  95 102  95 101


90  94  93  89  86  93  91  90  89  86  83  85  87  74  72  73  71
82  83  88  90 102 106 111 110 116 118 125 128 126 127 136 136 136
141 135 142 141 131 130 121 122 129 122 130 125 123 131


125 131 129 126 131 130 128 128 135 133 129 132 129 127 124 133 127
126 128 127 129 126 127 127 126 124 124 136 129 129 128 127 129 134
129 136 135 134 129 130 131 136 136 136 132 132 132 127


131 134 134 161 181 189 198 201 206 207 206 210 212 213 211 213 215
210 204 197 183 156 131  96  77  77  71  74  76  83  73  85  87  89
91  88  90  90  94  93 101  93  88  88  92  96  92  87


94  93  98 101  92  90  94  90  89  92  94  88  93  88  91  90  95
93 100  86  93  97  94  97  96  97  93  95  94  94  87  86 100  92
97  96  95 101  95  97  93  89 101  95  97  97  88  94


82  91  84  88  89  84  91  93 107 139 144 147 149 148 130  99 137
137 137 136 138 129 138 134 140 136 135 134 130 139 135 129 134 131
138 139 127 132 129 134 131 133 130 125 130 129 127 131


127 134 128 135 133 144 135 142 143 142 143 150 148 154 153 149 149
149 143 154 145 142 135 133 118 120 127 110  98  82  86  74  63  65
63  61  55  67  72  66  73  69  73  74  73  73  70  78


80  76  75  73  72  80  80  78  77  75  75  76  78  78  75  78  78
79  78  78  73  75  73  75  79  79  89  77  81  86  90  86  78  89
89  92  95  92  90  92  91 101  97  92  89  94 104 100
```

```
101  96 104  98  96  98 100  95 106 108 102 103 100 100  96  99 104
 91  98  95 105 101  98 101  97 102  97  97 105 104 105 103  95 102
 97 105 102 102 103 103  99 103  96 106 102  99 102 101


106 100 104 103 101 102 100 100 100 104 108 103 104  99 104  99 105
106 101 105 105 105 101 109 106 104 102 105 104 105 102 104 112 102
 99 110 104 101 100 102 102  98 100 103 103  99  98 100


101  97 107 105 104 106 101 100 100 101  99 102  97  98  99 103 105.
103 101 103 101 105 104 109 102  99 102 103  96  96 103 108 109 114
104 102 102  97  99  97 100 102  98 101 102 102  99  98


 99  97 100 102  93 100 105 105  99  97  96 102  96 101 104  94  95
103  99  95  99  95 102  93  97  99  92  96  95 102  95 101  90  94
 93  89  86  93  91  90  89  86  83  85  87  74  72  73


 71  82  83  88  90 102 106 111 110 116 118 125 128 126 127 136 136
136 141 135 142 141 131 130 121 122 129 122 130 125 123 131 125 131
129 126 131 130 128 128 135 133 129 132 129 127 124 133


127 126 128 127 129 126 127 127 126 124 124 136 129 129 128 127 129
134 129 136 135 134 129 130 131 136 136 136 132 132 132 127 131 134
134 161 181 189 198 201 206 207 206 210 212 213 211 213


215 210 204 197 183 156 131  96  77  77  71  74  76  83  73  85  87
 89  91  88  90  90  94  93 101  93  88  88  92  96  92  87  94  93
 98 101  92  90  94  90  89  92  94  88  93  88  91  90


 95  93 100  86  93  97  94  97  96  97  93  95  94  94  87  86 100
 92  97  96  95 101  95  97  93  89 101  95  97  97  88  94  82  91
 84  88  89  84  91  93 107 139 144 147 149 148 130  99


137 137 137 136 138 129 138 134 140 136 135 134 130 139 135 129 134
131 138 139 127 132 129 134 131 133 130 125 130 129 127 131 127 134
128 135 133 144 135 142 143 142 143 150 148 154 153 149


149 149 143 154 145 142 135 133 118 120 127 110  98  82  86  74  63
 65  63  61  55  67  72  66  73  69  73  74  73  73  70  78  80  76
 75  73  72  80  80  78  77  75  75  76  78  78  75  78


 78  79  78  78  73  75  73  75  79  79  89  77  81  86  90  86  78
 89  89  92  95  92  90  92  91 101  97  92  89  94 104 100 101  96
104  98  96  98 100  95 106 108 102 103 100 100  96  99


104  91  98  95 105 101  98 101  97 102  97  97 105 104 105 103  95
102  97 105 102 102 103 103  99 103  96 106 102  99 102 101 106 100
104 103 101 102 100 100 100 104 108 103 104  99 104  99
```

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105 106 101 105 105 105 101 109 106 104 102 105 104 105 102 104 112
102  99 110 104 101 100 102 102  98 100 103 103  99  98 100 101  97
107 105 104 106 101 100 100 101  99 102  97  98  99 103


105 103 101 103 101 105 104 109 102  99 102 103  96  96 103 108 109
114 104 102 102  97  99  97 100 102  98 101 102 102  99  98  99  97
100 102  93 100 105 105  99  97  96 102  96 101 104  94


 95 103  99  95  99  95 102  93  97  99  92  96  95 102  95 101  90
 94  93  89  86  93  91  90  89  86  83  85  87  74  72  73  71  82
 83  88  90 102 106 111 110 116 118 125 128 126 127 136


136 136 141 135 142 141 131 130 121 122 129 122 130 125 123 131 125
131 129 126 131 130 128 128 135 133 129 132 129 127 124 133 127 126
128 127 129 126 127 127 126 124 124 136 129 129 128 127


129 134 129 136 135 134 129 130 131 136 136 136 132 132 132 127 131
134 134 161 181 189 198 201 206 207 206 210 212 213 211 213 215 210
204 197 183 156 131  96  77  77  71  74  76  83  73  85


 87  89  91  88  90  90  94  93 101  93  88  88  92  96  92  87  94
 93  98 101  92  90  94  90  89  92  94  88  93  88  91  90  95  93
100  86  93  97  94  97  96  97  93  95  94  94  87  86


100  92  97  96  95 101  95  97  93  89 101  95  97  97  88  94  82
 91  84  88  89  84  91  93 107 139 144 147 149 148 130  99 137 137
137 136 138 129 138 134 140 136 135 134 130 139 135 129


134 131 138 139 127 132 129 134 131 133 130 125 130 129 127 131 127
134 128 135 133 144 135 142 143 142 143 150 148 154 153 149 149 149
143 154 145 142 135 133 118 120 127 110  98  82  86  74


 63  65  63  61  55  67  72  66  73  69  73  74  73  73  70  78  80
 76  75  73  72  80  80  78  77  75  75  76  78  78  75  78  78  79
 78  78  73  75  73  75  79  79  89  77  81  86  90  86


 78  89  89  92  95  92  90  92  91 101  97  92  89  94 104 100 101
 96 104  98  96  98 100  95 106 108 102 103 100 100  96  99 104  91
 98  95 105 101  98 101  97 102  97  97 105 104 105 103


 95 102  97 105 102 102 103 103  99 103  96 106 102  99 102 101 106
100 104 103 101 102 100 100 100 104 108 103 104  99 104  99 105 106
101 105 105 105 101 109 106 104 102 105 104 105 102 104


112 102  99 110 104 101 100 102 102  98 100 103 103  99  98 100 101
 97 107 105 104 106 101 100 100 101  99 102  97  98  99 103 105 103
101 103 101 105 104 109 102  99 102 103  96  96 103 108
```

The picture representing the final image is shown in Fig. 4b.

Example 3 (Rotation of a picture)

The object picture is the same as in the Example 2. The required operation is the rotation by 90° in the counterclockwise direction (Fig. 5a represents the result of a rotation of the object picture).

The array of the gray levels of the rotated piece S' of the object picture is th⁻ following array 10.

```
 60  66  75  86 100 117 114 112 111 111 112 114 118 118 118 118 118
118 118 119 119 117 113 107  92  88  84  81  80  79  68  67  67  66
 65  65  80  84  88  92  95  99  97  97  97  97  97  97


 58  64  73  84  98 115 117 115 114 114 115 117 118 118 118 118 118
118 117 118 117 114 110 104  90  87  85  83  83  83  75  74  73  73
 72  71  80  83  86  90  93  96  97  97  97  97  97  97


 56  62  71  82  96 113 119 118 117 117 118 119 118 118 118 118 118
118 116 117 115 112 107 101  88  86  85  85  86  88  81  81  80  79
 79  78  80  83  86  89  91  94  97  97  97  97  97  97


 56  62  70  82  96 112 122 120 119 119 120 122 118 118 118 118 118
118 115 115 114 110 105  98  86  86  86  87  90  93  88  87  87  86
 85  85  81  83  86  88  91  93  97  97  97  97  97  97


 56  62  71  82  96 113 125 123 122 122 123 125 118 118 118 118 118
118 114 114 112 108 102  94  84  85  87  89  93  97  95  94  93  93
 92  91  83  85  87  89  91  93  97  97  97  97  97  97


 57  63  72  83  97 114 127 126 125 125 126 127 118 118 118 118 118
118 114 113 110 106  99  91  82  84  87  91  96 102 101 101 100  99
 99  98  86  87  89  91  92  94  97  97  97  97  97  97


 50  60  71  82  95 109 132 131 130 128 124 120 125 122 119 117 114
111 109 105 100  95  91  86  94  89  86  86  89  94  95 101 105 106
104 100 100  99  97  96  95  93  96  97  98  98  99 100


 48  58  68  80  92 105 130 130 130 128 126 123 124 121 118 116 113
110 108 103  98  94  89  84  89  80  73  69  68  69  93 100 104 105
104 100 100  99  98  96  95  94  94  95  96  96  97  98


 48  57  67  78  90 102 128 130 130 130 128 126 125 122 119 117 114
111 107 102  97  93  88  83  88  74  64  56  50  48  92  99 103 105
104 100 100  99  97  96  95  93  92  93  94  94  95  96


 48  57  67  77  88 101 129 130 132 132 132 130 127 125 122 119 117
114 107 102  97  93  88  83  90  72  58  46  36  30  90  97 102 104
104 101  99  97  96  95  93  92  90  91  92  92  93  94


 50  58  67  77  88 100 129 132 134 136 136 136 132 129 126 124 121
118 108 103  98  94  89  84  95  74  55  39  26  15  88  96 101 104
104 101  96  95  94  92  91  90  88  89  90  90  91  92


 52  60  68  78  88 100 130 134 138 140 141 142 138 135 133 130 127
125 109 105 100  95  91  86 104  79  56  36  19   4  86  95 100 104
104 102  93  92  91  89  88  87  86  87  88  88  89  90
```

```
 61  65  70  77  87  97 112 140 158 168 168 160 157 152 147 141 136
131 125 114 106  99  94  91 105  83  66  53  44  40  97 103 107 108
107 102  88  86  84  84  84  85  86  87  87  88  89  89


 65  69  75  82  91 102 126 154 173 183 184 176 180 175 170 166 161
156 144 131 120 111 103  97 105  85  70  59  53  51  97 103 106 107
105 100  88  86  84  84  84  85  82  83  84  84  85  86


 69  73  78  85  95 105 136 164 183 194 195 188 195 190 186 182 178
173 158 142 129 117 106  98 102  84  71  63  59  59  96 102 105 105
103  97  88  86  84  84  84  85  81  81  82  83  83  84


 71  75  81  88  97 108 140 169 189 200 202 195 201 198 194 190 186
183 167 148 132 117 104  93  96  81  70  64  62  65  95 100 102 102
 99  94  88  86  84  84  84  85  81  81  82  83  83  84


 73  77  83  90  99 110 141 170 190 202 204 198 200 197 194 190 187
184 170 149 130 113  97  83  88  75  67  63  63  68  92  97  99  98
 95  89  88  86  84  84  84  85  82  83  84  84  85  86


 74  78  83  91 100 111 137 166 187 199 202 196 191 188 185 183 180
177 168 144 123 103  84  68  77  66  60  58  61  68  89  93  95  94
 90  83  88  86  84  84  84  85  86  87  87  88  89  89


 83  86  93 103 117 134 181 189 190 186 175 158 174 169 165 161 157
152 116 119 116 105  87  62  73  67  65  66  71  80  78  81  83  86
 89  91  92  91  89  88  87  86  90  90  91  92  92  93


 80  84  91 102 116 134 169 178 181 178 168 152 161 158 155 151 148
145 124 128 124 114  96  71  74  68  66  67  72  80  77  80  83  85
 88  91  94  93  92  92  91  90  92  93  94  94  95  96


 76  80  87  98 113 131 161 172 176 174 166 151 157 155 152 150 147
145 136 139 136 125 107  82  76  70  68  69  74  82  79  82  85  87
 90  93  97  97  97  97  97  96  98  98  99 100 100 101


 69  74  82  93 108 127 158 170 175 174 168 154 161 159 158 156 155
153 150 153 150 139 121  96  78  72  70  72  76  85  84  87  89  92
 95  97 103 103 103 104 104 104 106 106 107 108 108 109


 61  66  74  86 102 121 159 172 179 180 174 162 173 172 171 171 170
169 166 170 166 156 138 113  82  76  74  75  80  88  91  94  97  99
102 105 110 111 112 112 113 114 116 117 118 118 119 120


 51  56  65  78  94 113 164 179 187 189 185 174 192 193 193 193 193
194 186 189 186 175 157 132  86  80  78  80  84  93 101 104 107 109
112 115 119 121 122 123 124 125 130 130 131 132 132 133
```

```
 55  48  50  61  81 110 126 156 179 193 200 198 191 191 191 190 188
185 188 187 184 178 169 158  88  82  80  81  86  94 111 118 123 127
128 128 129 130 132 133 134 136 142 144 146 148 150 152


 63  55  56  66  85 113 127 156 177 189 194 191 187 188 188 188 186
184 185 186 184 179 172 162 104  98  96  97 102 110 126 133 138 141
143 143 139 140 141 143 144 145 161 163 165 167 169 171


 70  62  62  71  89 116 130 156 175 186 189 184 183 185 185 185 184
182 182 184 184 180 174 166 121 115 113 114 119 128 141 148 153 156
158 157 152 153 154 156 157 158 178 180 182 184 186 188


 78  68  68  76  93 119 133 158 175 184 185 178 179 181 182 183 182
180 178 182 182 180 176 168 139 133 131 132 137 146 155 162 167 171
172 172 168 170 171 172 174 175 192 194 196 198 200 202


 85  75  73  81  97 122 137 160 175 183 182 173 176 178 180 180 180
179 173 178 180 179 176 170 158 152 150 151 156 164 170 177 182 185
187 187 189 190 191 193 194 195 203 205 207 209 211 213


 93  82  79  86 101 125 142 163 177 182 180 169 172 175 177 178 178
177 168 174 177 178 176 171 178 172 170 171 176 184 185 192 197 200
202 201 212 214 215 216 218 219 212 214 216 218 220 222


 83  74  73  78  91 111 153 156 158 160 161 160 150 151 152 152 153
154 151 152 152 153 154 154 160 166 173 179 185 192 187 198 207 214
219 223 221 221 220 220 220 220 230 227 224 221 217 214


 83  72  68  72  82 100 130 134 137 139 140 140 141 142 143 143 144
145 144 145 146 146 147 148 152 157 162 166 171 176 196 206 214 221
226 229 226 225 224 224 223 222 214 211 208 206 203 200


 87  74  68  69  78  93 113 117 121 123 125 125 133 134 134 135 136
136 138 138 139 140 140 141 146 149 152 155 157 160 192 202 210 216
220 223 219 218 217 216 215 213 196 193 191 189 187 185


 96  81  73  72  78  91 101 106 110 113 115 116 126 126 127 128 128
129 131 132 132 133 134 134 140 141 143 144 145 146 176 185 193 198
202 205 201 199 198 196 195 193 176 174 172 171 169 167


110  92  82  78  82  93  95 100 104 108 110 112 119 120 121 121 122
123 124 125 126 126 127 128 136 135 134 134 133 132 147 156 163 168
172 174 171 169 167 165 163 161 154 153 152 150 149 148


127 108  95  89  91 100  94  99 104 108 111 113 114 114 115 116 116
117 118 118 119 120 120 121 132 129 127 125 122 120 106 114 121 126
129 130 130 127 125 122 120 117 131 130 129 128 127 127
```

```
124 119 113 107 100  91 104 105 1C7 109 111 112 112 112 112 112 112
112 121 122 122 123 124 124 121 120 119 118 117 116 118 118 118 118
118 118 119 124 128 131 133 134 135 136 136 137 138 138


120 120 119 117 114 110 105 106 107 109 110 111 114 114 114 114 114
114 118 119 120 120 121 122 119 118 117 117 116 115 118 118 118 118
118 118 119 123 127 129 131 132 132 133 134 134 135 136


108 112 116 118 120 120 106 107 108 108 109 110 114 114 114 114 114
114 116 116 117 118 118 119 116 116 116 116 115 115 118 118 118 118
118 118 118 122 125 128 129 129 130 130 131 132 132 133


 87  96 103 110 116 121 107 107 108 108 109 109 112 112 112 112 112
112 113 114 114 115 116 116 114 114 114 114 115 115 118 118 118 118
118 118 118 122 124 126 127 127 127 128 128 129 130 130


 57  70  82  94 104 114 108 108 108 108 108 108 108 108 108 108 108
108 110 111 112 112 113 114 111 112 113 113 114 115 118 118 118 118
118 118 118 121 123 125 125 125 124 125 126 126 127 128


 18  36  52  68  83  97 109 109 108 108 107 107 102 102 102 102 102
102 108 108 109 110 110 111 109 110 111 112 113 114 118 118 118 118
118 118 118 120 122 123 123 122 122 122 123 124 124 125


 20  20  21  23  25  29  72  82  92 102 111 121 108 106 103 101  98
 96 102 102 103 104 104 105 107 109 110 110 109 107 113 112 112 113
115 118 121 121 121 121 121 121 115 115 115 115 115 115


 20  20  20  22  24  27  50  58  66  74  82  90 104 105 106 106 107
108 100 101 102 102 103 104 106 108 109 109 108 106 111 110 110 110
112 114 118 118 118 118 118 118 114 114 114 114 114 114


 21  20  20  21  23  25  33  39  46  52  58  64  93  96 100 104 108
111  99 100 100 101 102 102 104 106 107 107 106 104 110 108 107 108
109 111 115 115 115 115 115 115 113 113 113 113 113 113


 21  20  20  20  21  24  22  26  30  35  39  44  73  80  87  93 100
107  98  98  99 100 100 101 102 104 104 104 104 102 108 106 105 105
105 107 111 111 111 111 111 111 111 111 111 111 111 111


 22  20  19  19  20  22  15  18  21  23  26  29  45  55  65  75  85
 95  96  97  98  98  99 100  98 100 101 101 100  98 106 104 102 102
102 103 108 108 108 108 108 108 110 110 110 110 110 110


 22  20  19  18  19  20  14  15  16  17  18  19   9  22  35  49  62
 75  95  96  96  97  98  98  94  96  96  96  96  94 105 102 100  99
 99  99 105 105 105 105 105 105 109 109 109 109 109 109
```

The above rotation acts on the Taylor polynomials, representing the intermediate image, obtained in the Example 2, as follows: let the 6 x 6 pixel square segments, into which the original picture has been subdivided, be indexed by two indices i and j, in such a way that the middle segment has indices 0, 0. Denote the Taylor polynomial corresponding to the segment i, j by p_{ij}. Then:

a. The indices i, j of each p_{ij} are replaced by -j, i

b. x is replaced by y, and y by -x.

Using the notations already used in discussing processing, $F(p_{ij}(x,y)) = p_{-j, i}(y, -x)$.

The result of the application of the corresponding subroutine to the Taylor polynomials in the intermediate range, obtained in the Example 2, is the intermediate range of the rotated picture. Its part P' corresponding to the rotated piece S', is the following array 11.

0.29296875	-0.0078125	0.0078125	0.015625	-0.000000	0.015625
0.45703125	0.0312500	-0.0312500	0.000000	-0.000000	0.000000
0.45703125	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.43750000	-0.0234375	0.0234375	0.000000	-0.015625	0.000000
0.33203125	0.0156250	-0.0156250	0.000000	0.046875	0.000000
0.32031250	0.0781250	-0.0781250	0.000000	-0.000000	0.000000
0.33593750	0.0000000	-0.0000000	0.015625	-0.015625	0.015625
0.37500000	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.27734375	-0.0078125	0.0078125	0.015625	-0.015625	0.015625
0.51171875	0.0234375	-0.0234375	0.015625	0.031250	0.015625
0.46093750	0.0312500	-0.0312500	0.031250	-0.000000	0.031250
0.36718750	0.0000000	-0.0000000	0.015625	-0.000000	0.015625
0.21093750	-0.0937500	0.0937500	0.062500	-0.140625	0.062500
0.40234375	-0.0078125	0.0078125	0.000000	0.015625	0.000000
0.37109375	-0.0156250	0.0156250	-0.015625	-0.000000	-0.015625
0.35937500	-0.0234375	0.0234375	0.000000	-0.000000	0.000000
0.32031250	0.0312500	-0.0312500	-0.015625	-0.000000	-0.015625
0.75390625	0.0703125	-0.0703125	-0.078125	0.015625	-0.078125
0.73828125	0.0937500	-0.0937500	-0.140625	0.015625	-0.140625
0.48437500	0.0234375	-0.0234375	-0.093750	-0.093750	-0.093750
0.25781250	0.0000000	-0.0000000	-0.046875	0.078125	-0.046875
0.40234375	-0.0312500	0.0312500	-0.015625	-0.015625	-0.015625
0.32421875	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.31640625	0.0000000	-0.0000000	0.031250	-0.000000	0.031250
0.34765625	-0.0625000	0.0625000	-0.031250	0.015625	-0.031250
0.68359375	0.0000000	-0.0000000	0.078125	0.046875	0.078125
0.59765625	0.0703125	-0.0703125	0.140625	0.031250	0.140625
0.53906250	0.1640625	-0.1640625	0.046875	-0.000000	0.046875
0.26562500	0.0312500	-0.0312500	0.015625	-0.000000	0.015625
0.33984375	0.0546875	-0.0546875	0.046875	-0.000000	0.046875
0.38671875	0.0781250	-0.0781250	0.031250	0.015625	0.031250
0.39843750	0.0937500	-0.0937500	0.046875	-0.000000	0.046875
0.26171875	0.0625000	-0.0625000	0.000000	-0.031250	0.000000
0.70703125	-0.0156250	0.0156250	0.015625	-0.062500	0.015625
0.71875000	-0.0312500	0.0312500	0.000000	0.015625	0.000000
0.71093750	-0.0078125	0.0078125	-0.015625	0.046875	-0.015625
0.47656250	0.2109375	-0.2109375	0.015625	-0.000000	0.015625
0.63281250	0.1718750	-0.1718750	0.000000	-0.000000	0.000000
0.63671875	0.1953125	-0.1953125	0.062500	-0.000000	0.062500
0.74218750	0.1640625	-0.1640625	-0.046875	-0.000000	-0.046875
0.26562500	0.0390625	-0.0390625	0.078125	-0.078125	0.078125
0.44921875	-0.1250000	0.1250000	0.093750	0.015625	0.093750
0.51171875	-0.0859375	0.0859375	0.015625	-0.000000	0.015625
0.53125000	-0.0781250	0.0781250	0.000000	-0.000000	0.000000
0.57812500	-0.1171875	0.1171875	0.015625	-0.062500	0.015625
0.80468750	-0.2031250	0.2031250	-0.218750	-0.015625	-0.218750
0.81250000	-0.2265625	0.2265625	-0.203125	-0.015625	-0.203125
0.70703125	-0.2187500	0.2187500	-0.031250	0.015625	-0.031250
0.43750000	-0.1171875	0.1171875	-0.156250	0.156250	-0.156250
0.41796875	0.0000000	-0.0000000	0.000000	-0.015625	0.000000
0.43750000	-0.0234375	0.0234375	-0.031250	-0.000000	-0.031250
0.44921875	-0.0312500	0.0312500	0.000000	-0.000000	0.000000
0.44531250	-0.0156250	0.0156250	0.000000	0.015625	0.000000
0.45703125	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.49218750	-0.0156250	0.0156250	0.000000	-0.015625	0.000000
0.50781250	-0.0312500	0.0312500	0.000000	-0.000000	0.000000
0.07421875	-0.0078125	0.0078125	0.000000	-0.015625	0.000000
0.15234375	-0.1875000	0.1875000	0.093750	-0.062500	0.093750
0.37500000	-0.1406250	0.1406250	-0.140625	0.109375	-0.140625
0.38671875	-0.0156250	0.0156250	0.000000	-0.000000	0.000000
0.41015625	-0.0312500	0.0312500	-0.015625	-0.000000	-0.015625
0:41015625	-0.0312500	0.0312500	0.000000	-0.015625	0.000000
0.43750000	-0.0390625	0.0390625	0.000000	-0.000000	0.000000
0.43359375	-0.0156250	0.0156250	0.000000	-0.000000	0.000000

The final image, produced from the data rotated in a compressed form, is shown in Fig. 5b.

Example 4 (Producing a negative picture)

The object picture is the same as in the Example 2. It is required to produce a negative of this picture. Under this operation each gray level value z must be replaced by $z' = 255 - z$.

The negative of the original picture is shown in Fig. 6a. The array S" of the gray levels, corresponding to the negative of the piece S, is the following.

```
158 158 158 158 158 158 155 157 159 161 163 165 166 169 171 171 169
166 162 159 154 146 135 122 103  84  67  53  42  33  41  55  70  88
107 128 117 119 122 125 127 130 140 141 142 144 145 146


158 158 158 158 158 158 156 158 160 162 164 166 166 170 172 172 170
166 163 160 155 147 136 123 105  86  69  55  44  35  38  52  68  86
106 128 117 120 123 125 128 131 140 141 142 144 145 146


158 158 158 158 158 158 157 159 161 163 165 167 167 171 172 172 171
167 163 161 155 147 137 123 107  88  71  57  46  37  34  49  66  84
105 127 118 121 123 126 129 131 140 141 142 144 145 146


158 158 158 158 158 158 157 159 161 163 165 167 168 171 173 173 171
168 164 161 156 148 137 124 109  90  73  59  48  39  31  47  64  83
103 126 119 121 124 127 129 132 140 141 142 144 145 146


158 158 158 158 158 158 158 160 162 164 166 168 168 172 174 174 172
168 165 162 157 149 138 125 111  92  75  61  50  41  28  44  62  81
102 125 119 122 125 127 130 133 140 141 142 144 145 146


158 158 158 158 158 158 159 161 163 165 167 169 169 173 174 174 173
169 165 163 157 149 139 125 113  94  77  63  52  43  25  41  59  79
101 124 120 123 125 128 131 133 140 141 142 144 145 146


156 159 161 162 162 161 162 161 162 163 165 168 170 170 170 170 170
170 169 165 159 151 141 130 119 110  97  80  60  36  35  33  42  62
 94 138 121 123 126 128 130 133 134 137 140 144 147 150


160 162 164 164 164 163 160 160 160 162 164 167 171 171 171 171 171
171 168 164 158 151 142 131 121 111  98  81  61  37  35  32  40  60
 92 135 122 124 126 128 130 132 134 137 140 144 147 150


163 165 166 167 166 164 159 159 159 160 163 166 171 171 171 171 171
171 167 163 158 151 143 132 122 112  99  83  62  39  35  31  39  59
 90 133 124 126 127 129 130 132 134 137 140 144 147 150


167 169 169 169 168 166 158 157 158 159 161 164 171 171 171 171 171
171 166 163 158 152 143 133 123 114 101  84  64  40  35  31  38  57
 88 130 127 128 130 131 132 133 134 137 140 144 147 150


171 172 172 172 170 168 156 156 156 158 160 163 169 169 169 169 169
169 164 162 158 152 144 134 125 115 102  85  65  41  34  30  37  56
 86 128 131 132 133 133 134 135 134 137 140 144 147 150


175 175 175 174 172 169 155 155 155 156 159 162 167 167 167 167 167
167 163 161 158 152 145 136 126 116 103  87  66  43  34  29  36  54
 84 125 136 136 137 137 137 137 134 137 140 144 147 150
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190 184 177 170 164 157 155 155 155 154 154 153 153 155 158 161 166
172 164 164 162 158 150 140 127 112  98  83  68  54  32  26  32  50
 81 125 137 137 137 137 137 137 137 141 144 148 152 156


190 183 176 170 163 156 151 151 151 151 151 151 148 150 152 156 160
165 166 167 165 160 153 143 127 112  97  83  68  53  36  29  35  53
 83 126 137 137 137 137 137 137 140 143 146 150 153 156


189 182 176 169 162 156 149 150 150 151 151 151 147 148 150 153 157
161 169 170 168 163 156 146 128 114  99  84  70  55  41  34  39  57
 87 129 137 137 137 137 137 137 142 145 147 150 153 156


188 182 175 168 162 155 150 151 152 153 154 155 148 149 150 153 156
160 172 172 170 166 158 148 132 117 102  88  73  58  48  41  45  62
 92 134 137 137 137 137 137 137 143 145 148 150 153 155


188 181 174 168 161 154 154 155 156 158 159 160 152 152 153 155 158
162 174 175 173 168 161 151 137 122 107  93  78  63  57  49  53  70
 99 141 137 137 137 137 137 137 143 145 147 149 151 153


187 180 174 167 160 154 160 162 163 165 167 169 158 158 159 160 163
166 177 178 176 171 164 154 144 129 114 100  85  70  68  59  63  79
108 149 137 137 137 137 137 137 142 144 145 147 149 150


176 172 167 162 158 153 161 186 207 225 240 251 215 204 196 190 187
187 175 175 173 170 167 162 161 145 127 109  91  71  63  79  95 109
123 135 139 140 140 140 140 141 148 149 151 153 157 161


175 172 169 165 162 159 166 187 205 219 229 236 211 202 196 193 192
194 184 183 181 179 175 171 169 153 136 118  99  79  70  84  98 110
122 133 138 139 140 140 141 142 146 147 149 151 155 159


174 172 170 168 166 164 169 186 199 209 216 219 202 196 192 191 192    ⦂
197 189 188 186 183 180 175 174 158 141 123 104  84  76  89 100 111
121 130 137 138 139 141 142 143 145 146 148 151 154 159


171 170 170 169 168 168 169 182 191 197 200 199 189 185 184 185 188
195 190 189 187 185 181 177 175 159 142 124 105  85  82  93 103 112
121 128 136 138 139 141 142 144 145 146 148 151 154 159


167 168 169 169 170 171 166 175 181 183 181 176 172 170 171 174 180
189 188 187 185 183 179 175 173 157 140 122 103  83  89  98 106 114
120 126 135 137 139 141 143 145 146 147 149 151 155 159


163 165 167 169 171 173 161 166 167 165 160 151 150 150 153 159 167
178 182 181 179 177 173 169 167 151 134 116  97  77  95 103 109 115
119 123 134 136 139 141 144 146 148 149 151 153 157 161
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148 151 154 157 161 164 169 171 172 172 171 169 164 158 157 162 172
187 193 184 173 159 142 123  97  93  89  87  85  84 101 107 114 121
127 134 131 133 136 139 141 144 150 151 153 154 155 157


142 145 148 150 153 156 164 166 167 167 166 164 161 152 149 151 158
171 168 159 148 134 117  98  86  83  81  79  79  79 101 108 115 121
128 135 131 134 137 139 142 145 151 152 153 155 156 157


138 141 143 145 147 149 160 161 162 162 161 160 156 144 138 138 142
152 150 141 130 116  99  80  77  76  75  75  76  77 102 109 115 122
129 135 132 135 137 140 143 145 151 153 154 155 157 158


136 138 140 141 143 145 155 157 158 158 157 155 149 135 126 123 125
132 139 131 119 105  89  69  71  71  71  73  75  78 103 109 116 123
129 136 133 135 138 141 143 146 152 153 155 156 157 159


136 137 138 140 141 142 150 152 153 153 152 150 141 124 113 107 106
111 136 127 116 102  85  66  68  69  71  73  77  81 103 110 117 123
130 137 133 136 139 141 144 147 153 154 155 157 158 159


137 138 139 140 141 141 146 147 143 148 147 146 130 111  97  88  85
 87 139 131 119 105  89  69  67  70  73  77  82  87 104 111 117 124
131 137 134 137 139 142 145 147 153 155 156 157 159 160


137 137 137 137 137 137 144 145 144 141 137 130 124  99  82  72  71
 78 103 110 110 102  86  61  70  71  73  75  76  78 101 110 119 126
132 138 143 141 141 143 147 153 159 147 144 148 160 180


137 137 137 137 137 137 141 142 141 138 134 128 119  94  77  69  68
 75  98 107 108 100  85  62  67  69  71  73  75  77 102 111 119 127
133 139 143 141 141 143 147 153 157 148 147 155 170 193


137 137 137 137 137 137 138 139 138 136 131 125 114  89  73  65  65
 72  94 104 105  99  84  62  65  67  70  72  75  77 103 112 120 127
134 139 143 141 141 143 147 153 154 149 151 162 180 206


137 137 137 137 137 137 136 137 136 133 129 122 108  85  69  61  61
 70  90 100 103  97  84  62  64  67  70  73  75  78 103 112 121 128
134 140 143 141 141 143 147 153 152 149 155 168 190 220


137 137 137 137 137 137 133 134 133 130 126 120 103  80  65  57  58
 67  86  97 100  96  83  62  64  67  70  74  77  80 104 113 121 129
135 141 143 141 141 143 147 153 149 150 159 175 200 233


137 137 137 137 137 137 130 131 130 128 123 117  98  75  60  54  55
 64  81  94  98  94  82  63  64  68  72  76  79  83 105 114 122 129
136 141 143 141 141 143 147 153 147 151 162 182 210 246
```

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141 138 136 133 130 128 135 132 129 125 119 113  95  79  67  60  57
 59  97 103 104 101  93  81  57  64  71  77  82  86  95 115 130 139
143 142 143 144 145 146 147 148 134 165 191 211 226 236


143 140 137 135 132 129 131 129 127 123 119 114  87  71  60  53  51
 53  80  87  89  87  81  70  55  61  66  70  73  75  94 115 130 140
145 144 144 145 146 146 147 148 144 173 197 216 229 237


144 141 138 136 133 130 127 127 125 123 119 115  87  72  61  55  53
 56  69  77  81  81  75  66  62  66  69  71  72  73  95 116 132 142
147 147 146 146 147 147 147 147 153 181 203 220 232 238


144 141 138 136 133 130 125 125 125 123 121 117  97  82  72  66  65
 68  65  74  79  80  76  68  76  78  80  80  80  78  97 118 134 145
151 151 148 148 147 147 147 147 163 189 209 225 234 239


143 140 137 135 132 129 124 125 125 125 123 121 115 101  91  86  85
 89  66  77  83  85  83  76  99  99  99  97  95  92  99 121 138 149
155 156 150 149 148 148 147 146 173 197 216 229 237 240


141 138 136 133 130 128 123 125 127 127 126 125 143 129 119 115 114
118  74  86  94  97  96  91 129 128 125 122 118 113 102 125 142 154
160 161 151 150 149 148 147 146 183 205 222 233 240 241


138 140 142 143 142 141 146 150 153 154 155 155 158 153 150 147 145
144 121 121 124 128 134 142 145 142 139 136 133 130 144 155 162 164
162 155 164 145 135 134 141 158 226 228 230 231 233 235


155 157 159 159 159 158 160 163 165 167 167 167 168 164 160 158 156
155 138 139 142 147 153 161 174 170 166 162 158 154 164 173 177 177
173 164 155 141 135 139 151 172 230 231 232 234 235 236


169 171 173 173 173 172 173 175 177 178 178 177 178 173 170 167 165
164 152 153 157 162 169 177 194 189 184 179 174 169 177 183 186 183
177 166 148 138 137 145 161 187 232 233 234 235 236 237


180 182 184 185 184 183 184 187 188 188 188 187 185 180 177 174 172
172 162 164 168 173 181 190 205 199 193 187 182 176 182 187 187 182
173 160 142 136 139 152 173 203 234 235 235 235 236 236


189 191 193 193 193 192 195 197 198 198 197 195 190 186 182 180 178
177 169 171 175 181 189 199 207 200 193 187 180 173 181 183 181 174
163 147 136 135 143 159 185 219 235 235 235 235 235 235


195 197 199 199 199 198 205 207 207 207 205 203 194 190 186 184 182
181 172 175 179 186 194 204 200 192 185 177 170 162 172 172 168 159
145 128 131 135 147 168 198 237 235 235 234 234 233 233
```

The above operation on Taylor polynomials is the following:

$F(a_0 + a_1 x + a_2 y + a_{11} x^2 + 2a_{12} xy + a_{22} y^2) =$

$1 - a_0 - a_1 x - a_2 y - a_{11} x^2 - 2a_{12} xy - a_{22} y^2$

(in the same rescaling as above).

The corresponding subroutine, applied to the Taylor polynomials of the intermediate image obtained in the Example 2, gives the intermediate image of the negative. The part of the polynomials array P", corresponding to the piece S" of the negative, is the following.

0.62500000	-0.0000000	-0.0000000	-0.000000	-0.000000	-0.000000
0.64062500	0.0078125	0.0234375	-0.000000	-0.000000	-0.000000
0.68359375	0.0078125	-0.0000000	-0.000000	-0.000000	-0.031250
0.60156250	0.0078125	-0.0937500	-0.000000	-0.000000	-0.046875
0.25781250	0.0234375	-0.1640625	-0.000000	-0.000000	0.046875
0.29296875	-0.0234375	0.2187500	-0.000000	0.015625	0.031250
0.49218750	0.0078125	0.0312500	-0.000000	-0.000000	-0.000000
0.56640625	-0.0000000	0.0156250	-0.000000	-0.000000	-0.000000
0.66406250	0.0312500	-0.0000000	-0.000000	-0.015625	-0.015625
0.62890625	-0.0156250	0.0156250	-0.000000	-0.000000	0.015625
0.67578125	-0.0078125	-0.0000000	-0.015625	-0.000000	-0.000000
0.61328125	-0.0000000	-0.0781250	-0.000000	0.015625	-0.031250
0.36328125	0.0156250	-0.1953125	-0.000000	-0.000000	-0.062500
0.18750000	-0.0156250	0.2265625	-0.000000	-0.015625	0.203125
0.50781250	0.0234375	0.0156250	0.015625	-0.015625	-0.000000
0.56250000	-0.0000000	0.0390625	-0.000000	-0.000000	-0.000000
0.67968750	-0.0078125	-0.0781250	-0.000000	-0.000000	-0.000000
0.59765625	0.0234375	0.0078125	0.046875	0.015625	-0.000000
0.59765625	-0.0000000	0.0312500	0.046875	-0.015625	0.015625
0.66015625	0.0312500	-0.0546875	-0.000000	-0.000000	-0.046875
0.36718750	0.0390625	-0.1718750	0.031250	-0.000000	-0.000000
0.19531250	0.0703125	0.2031250	0.031250	-0.015625	0.218750
0.54296875	-0.0000000	-0.0000000	-0.000000	-0.000000	-0.000000
0.58984375	-0.0000000	0.0312500	-0.015625	-0.015625	-0.000000
0.66796875	0.0078125	-0.0156250	-0.015625	0.046875	-0.000000
0.78906250	-0.1171875	0.0937500	-0.046875	-0.140625	-0.062500
0.74218750	-0.0859375	-0.0000000	-0.078125	0.078125	0.046875
0.73437500	0.0156250	-0.0312500	-0.062500	-0.000000	-0.015625
0.52343750	0.0156250	-0.2109375	-0.062500	-0.000000	-0.015625
0.42187500	0.0234375	0.1171875	-0.000000	-0.062500	-0.015625
0.55468750	-0.0000000	0.0156250	-0.000000	0.015625	-0.000000
0.58984375	-0.0000000	0.0312500	0.015625	-0.000000	0.015625
0.56250000	-0.0390625	0.0234375	0.031250	-0.015625	-0.000000
0.63281250	-0.0546875	-0.0000000	-0.000000	-0.000000	-0.015625
0.51562500	-0.1562500	-0.0234375	-0.031250	-0.093750	0.093750
0.46093750	-0.1250000	-0.1640625	0.125000	-0.000000	-0.046875
0.28906250	-0.0312500	0.0078125	0.046875	0.046875	0.015625
0.46875000	0.0078125	0.0781250	-0.000000	-0.000000	-0.000000
0.55078125	0.0078125	0.0312500	-0.000000	-0.000000	-0.000000
0.61328125	0.0078125	0.0156250	-0.000000	-0.000000	-0.000000
0.54296875	-0.0000000	-0.0000000	-0.000000	-0.000000	-0.000000
0.53906250	-0.0312500	-0.0312500	-0.000000	-0.000000	-0.031250
0.26171875	-0.0468750	-0.0937500	-0.000000	0.015625	0.140625
0.40234375	-0.0234375	-0.0703125	-0.000000	0.031250	-0.140625
0.28125000	-0.0000000	0.0312500	0.015625	0.015625	-0.000000
0.48828125	0.0078125	0.0859375	-0.000000	-0.000000	-0.015625
0.56250000	-0.0000000	0.0234375	-0.000000	-0.000000	0.031250
0.62500000	0.0625000	0.1406250	-0.000000	0.109375	0.140625
0.54296875	-0.0000000	-0.0312500	-0.015625	-0.000000	-0.000000
0.48828125	-0.0000000	-0.0234375	0.015625	0.031250	-0.015625
0.24609375	0.1250000	-0.0703125	0.156250	0.015625	0.078125
0.31640625	-0.0156250	-0.0000000	0.109375	0.046875	-0.078125
0.29296875	0.1171875	0.0156250	0.140625	-0.062500	-0.015625
0.55078125	0.0312500	0.1250000	0.015625	0.015625	-0.093750
0.58203125	0.0078125	-0.0000000	-0.000000	-0.015625	-0.000000
0.84765625	0.0625000	0.1875000	-0.000000	-0.062500	-0.093750
0.70703125	0.1328125	0.0078125	-0.046875	-0.000000	-0.015625
0.72265625	0.1250000	0.0078125	-0.015625	-0.015625	-0.015625
0.67968750	0.0859375	-0.0312500	-0.031250	-0.000000	0.015625
0.65234375	0.1328125	0.0625000	-0.062500	0.015625	0.031250
0.73828125	0.1015625	-0.0625000	-0.156250	-0.031250	-0.000000
0.73437500	-0.0000000	-0.0390625	-0.125000	-0.078125	-0.078125
0.56250000	0.0546875	0.1171875	0.015625	0.156250	0.156250
0.92578125	0.0078125	0.0078125	-0.015625	-0.015625	-0.000000

The final image produced from the intermediate negative image, obtained as above, is shown in Fig. 6b.

While a number of embodiments of the invention have been discussed and illustrated, it will be understood that the invention may be carried out in a number of ways and with many modifications, adaptations, and variations, by persons skilled in the art, without departing from its spirit and from the scope of the appended claims.

CLAIMS

1 - Process for the production of images of objects, as hereinbefore defined, comprosing the steps of:

(1) Approximating the object by a model comprising at least one differentiable component.

(2) Establishing the maximum allowable error ε and the degree k of the polynomials by which the differentiable component(s) of the model are to be approximated.

(3) Constructing a grid of a suitable pitch h.

(4) Computing the coefficients of the Taylor polynomials of the aforesaid differentiable component(s) at selected points of said grid.

2 - Process according to claim 1, wherein the object is defined in a space having more than three dimensions.

3 - Process according to claim 1, wherein the object is a line.

4 - Process according to claim 1, wherein the object is a surface.

5 - Process according to claim 1, wherein the object is a solid.

6 - Process according to claim 1, wherein the model further comprises at least one non-differentiable component.

7 - Process according to claim 1, comprising carrying out the said steps at least in part concurrently.

8 - Process according to claim 1, wherein the object is defined by data which are values and/or relationships embodied in physical entities.

9 - Process according to claim 8, comprising the preliminary step of storing the data defining the object in an electronic memory.

10 - Process according to claim 1, comprising determining the parameters of the components of the model by minimizing a quantity representing an error

11 - Process according to claim 10, wherein the quantity representing an error is the quadratical error.

12 - Process according to claim 1, wherein the non-differentiable component(s) of the model embody the same discontinuities as the object, and the differentiable component(s) represent the deviations of the object lfrom the non-differentiable component.

`13 - Process according to claim 12, wherein the model has the form:

(1) $\Phi(x) = Hx_0,a,b,c,d(x) + \phi(x)$

wherein H is defined by $H(x) = a(x-x_0) + b$, if $x \geq x_0$ or $H(x) = c(x-x_0) + d$, if x is less than x_0.

1 3 - Process according to claim 1, wherein the model is a differentiable function of another function which embodies the non-differentiable characteristics of the object.

1 4 - Process according to claim 1, wherein each grid pitch is. calculatedfrom the formula

(3) $CMh^{k+1} \leq \varepsilon$

wherein $C = 1/(k+1)!$ and M is the maximum, at each grid point, of the absolute value of the derivatives of degree k+1 of the differentiable component or components of the model.

1 5- Process according to claim 1, further comprising constructing an adjusted image line by applying to each differentiable component the Whitney subroutine, and minimizing the quntity W thus computed, under such constraints that the results of the minimization do not deviate from the initial data by more than the allowed error.

1 6 - Process according to claim 1, further comprising rounding off the coefficients of the Taylor polynomials to a maximum allowable error greater than the original one.

18 – Process according to claim 1, further comprising separating a temporary image into components of increasing fineness, constructing a grid which is sparser than the one used for obtaining said image and the pitch of which is determined by the resolution required by the lowest fineness of said components, obtaining thereforom a second temporary image, subtracting said second temporary image from the original one to obtain a first residual image, and repeating the same steps for successively finer components, correspondingly obtaining successive residual images, whereby to compute coefficients of Taylor polynomials on several grids having increasingly higher resolutions.

19 - Process according to claim 1, further comprising applying to the coefficients of the Taylor polynomials any desired known encoding method.

20 - Process according to claim 1, further comprising applying to any data obtained in carrying out the process any desired known encoding method.

21 - Process according to claim 1, further comprising constructing a final image by a procedure comprising the steps of dividing the domain, in which the temporary image has been defined, into possibly overlapping regions by means of a grid, each region being a portion of the grid around a grid node, and constructing curves representing the Taylor polynomials of degree k from the coefficients defining the temporary image at each grid node.

22 - Process according to claim 1, further comprising processing the obtained data, representing an intermediate image, by applying thereto an operator, whereby to obtain an image representing an object which is the result of applying to the original object the said operator.

COMPRESSED IMAGE PRODUCTION, STORAGE, TRANSMISSION AND PROCESSING

ABSTRACT

Images of objects are produced by:

(1) Approximating the object by a model comprising at least one differentiable component.

(2) Establishing the maximum allowable error ε and the degree k of the polynomials by which the differentiable component(s) of the model are to be approximated.

(3) Constructing a grid of a suitable pitch h.

(4) Computing the coefficients of the Taylor polynomials of the aforesaid differentiable component(s) at selected points of said grid.

2 - Process according to claim 1, wherein the object is defined in a space having more than three dimensions.

3 - Process according to claim 1, wherein the object is a line.

4 - Process according to claim 1, wherein the object is a surface.

5 - Process according to claim 1, wherein the object is a solid.

6 - Process according to claim 1, wherein the model further comprises at least one non-differentiable component.

7 - Process according to claim 1, comprising carrying out the said steps at least in part concurrently.

8 - Process according to claim 1, wherein the object is defined by data which are values and/or relationships embodied in physical entities.

9 - Process according to claim 8, comprising the preliminary step of storing the data defining the object in an electronic memory.

10 - Process for the production of images of objects, according to the claim 1, wherein said second component of the model is defined by minimizing, by a predetermined subroutine, a quantity representing the deviation from the object of a model consisting of the first and second components.

11 - Process for the production of images of objects according to claim 1, wherein the data defining the object, the data defining the model, and the data defining the images, are digital data.

12 - Process according to claim 1, wherein the model has the form:

(1) $\Phi(x) = Hx_0,a,b,c,d(x) + \phi(x)$

wherein H is defined by $H(x) = a(x-x_0) + b$, if $x \geq x_0$ or $H(x) = c(x-x_0) + d$, if x is less than x_0.

1 3 - Process according to claim 1, wherein the model is a differentiable function of another function which embodies the non-differentiable characteristics of the object.

1 4 - Process according to claim 1, wherein each grid pitch is calculatedfrom the formula

(3) $CMh^{k+1} \leq \epsilon$

wherein $C = 1/(k+1)!$ and M is the maximum, at each grid point, of the absolute value of the derivatives of degree k+1 of the differentiable component or components of the model.

1 5- Process according to claim 1, further comprising constructing an adjusted image line by applying to each differentiable component the Whitney subroutine, and minimizing the quntity W thus computed, under such constraints that the results of the minimization do not deviate from the initial data by more than the allowed error.

1 6 - Process according to claim 1, further comprising rounding off the coefficients of the Taylor polynomials to a maximum allowable error greater than the original one.

17 – Process according to claim 1, further comprising separating a temporary image into components of increasing fineness, constructing a grid which is sparser than the one used for obtaining said image and the pitch of which is determined by the resolution required by the lowest fineness of said components, obtaining thereforom a second temporary image, subtracting said second temporary image from the original one to obtain a first residual image, and repeating the same steps for successively finer components, correspondingly obtaining successive residual images, whereby to compute coefficients of Taylor polynomials on several grids having increasingly higher resolutions.

18 - Process according to claim 1, further comprising applying to the coefficients of the Taylor polynomials any desired known encoding method.

19 - Process according to claim 1, further comprising applying to any data obtained in carrying out the process any desired known encoding method.

20 - Process according to claim 1, further comprising constructing a final image by a procedure comprising the steps of dividing the domain, in which the temporary image has been defined, into possibly overlapping regions by means of a grid, each region being a portion of the grid around a grid node, and constructing curves representing the Taylor polynomials of degree k from the coefficients defining the temporary image at each grid node.
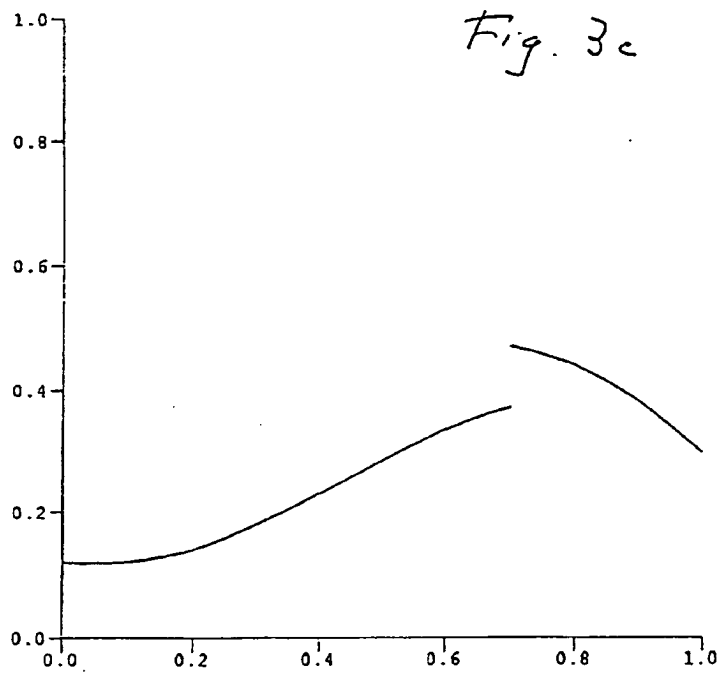
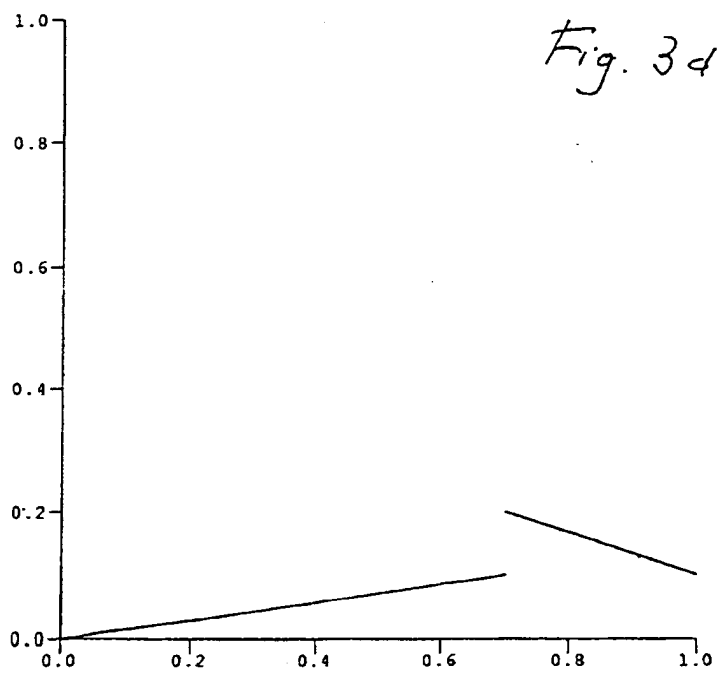Fig. 16
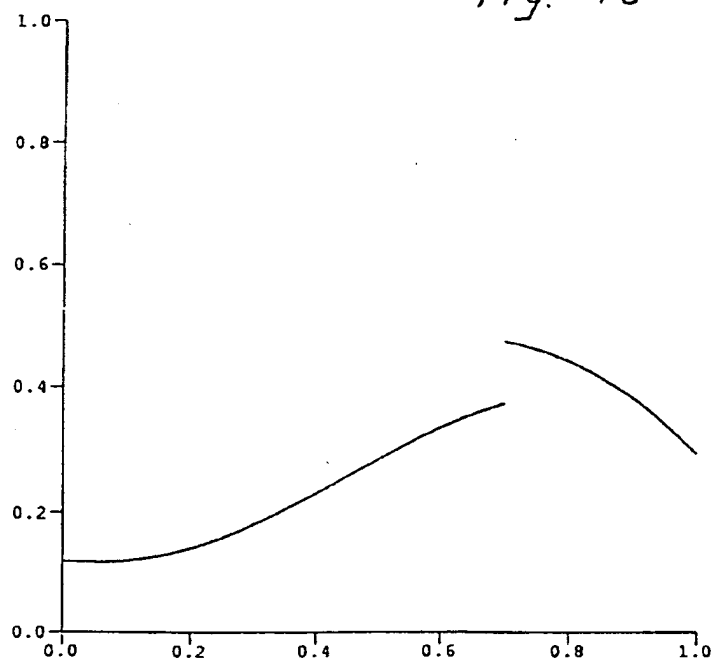
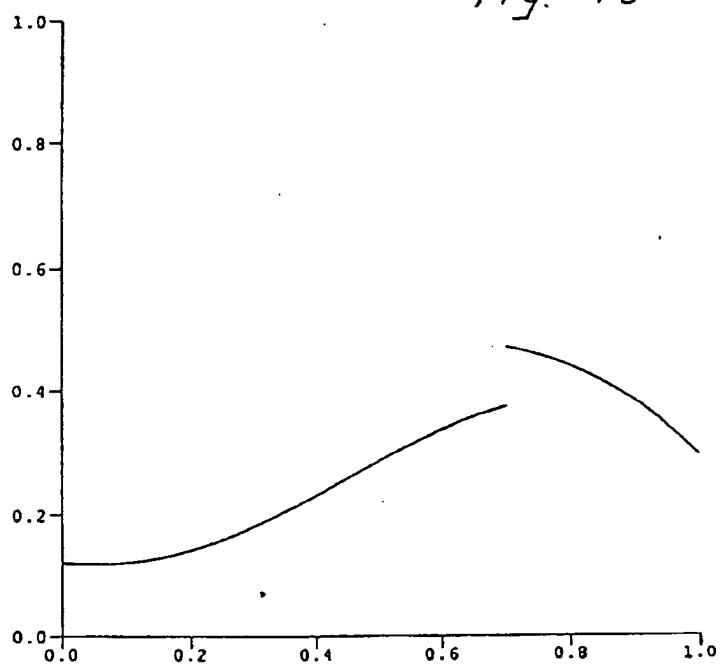Fig. 2a
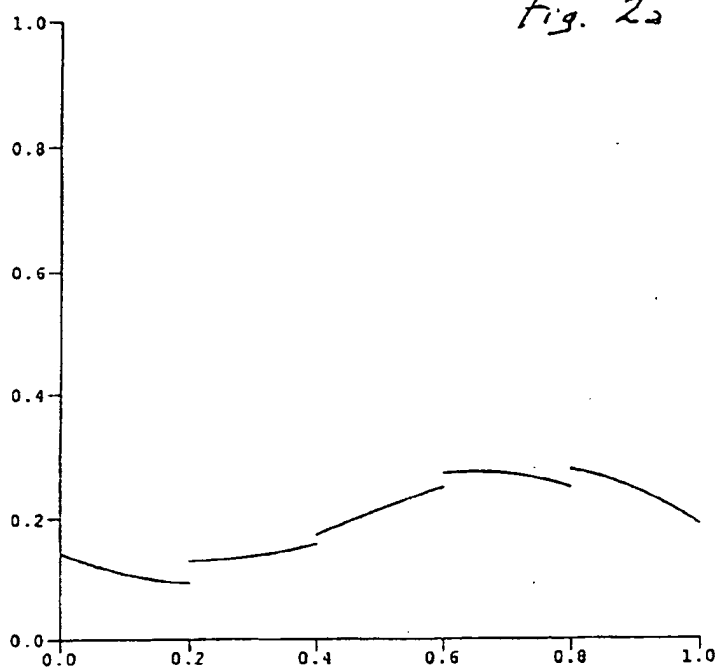
Fig. 26
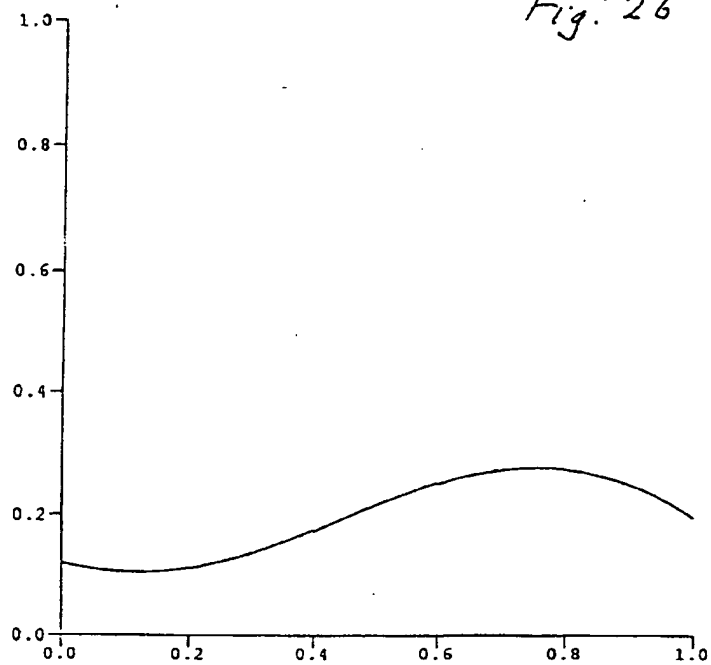
Fig. 3a

Fig. 36

Fig. 3c

Fig. 3d

Fig. 4a
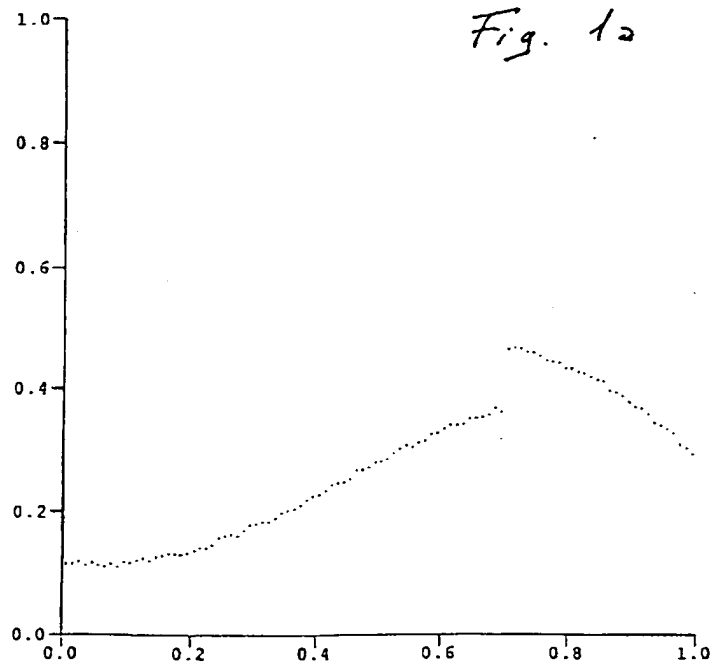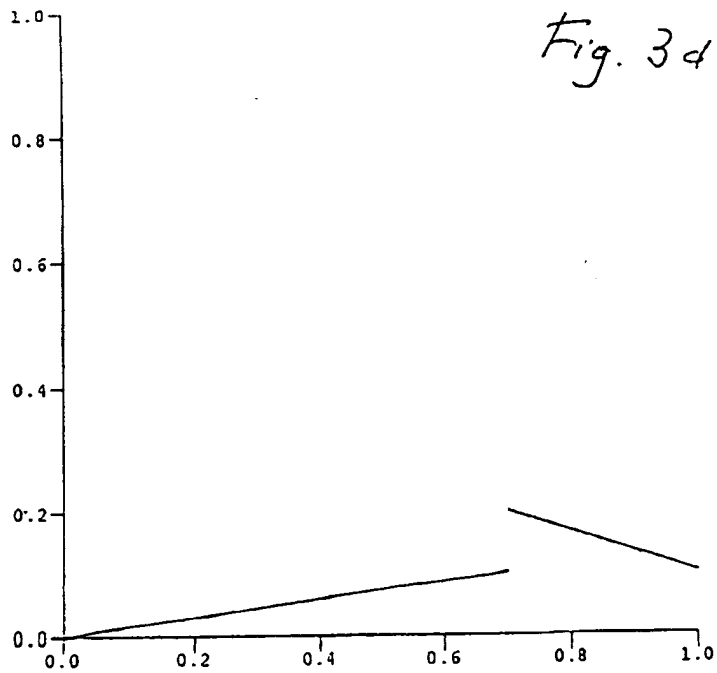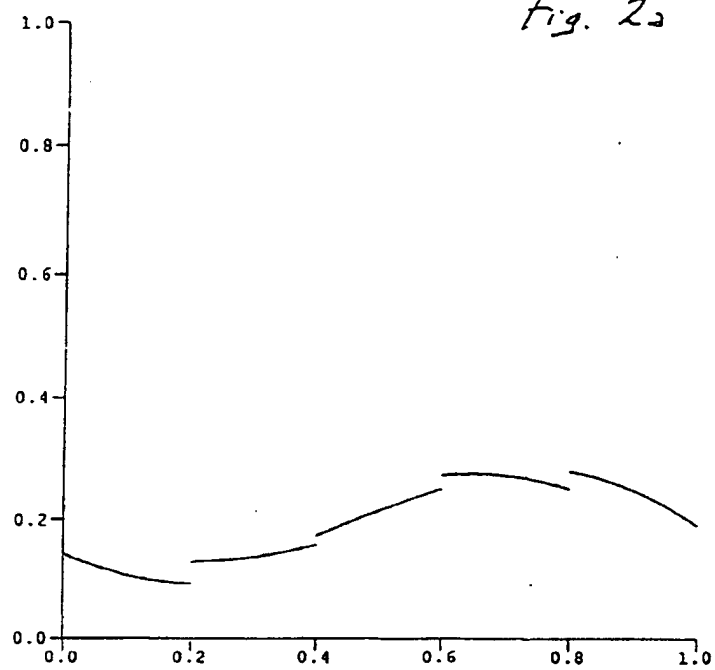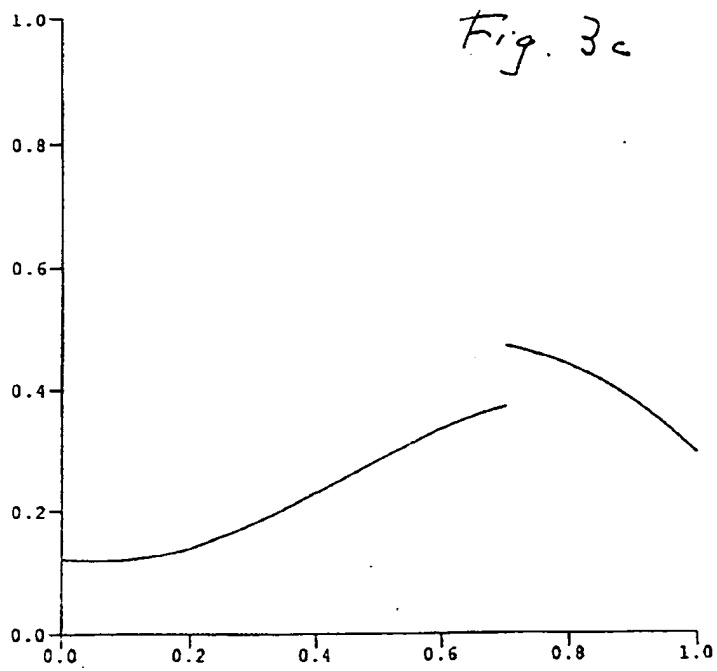
Fig. 46
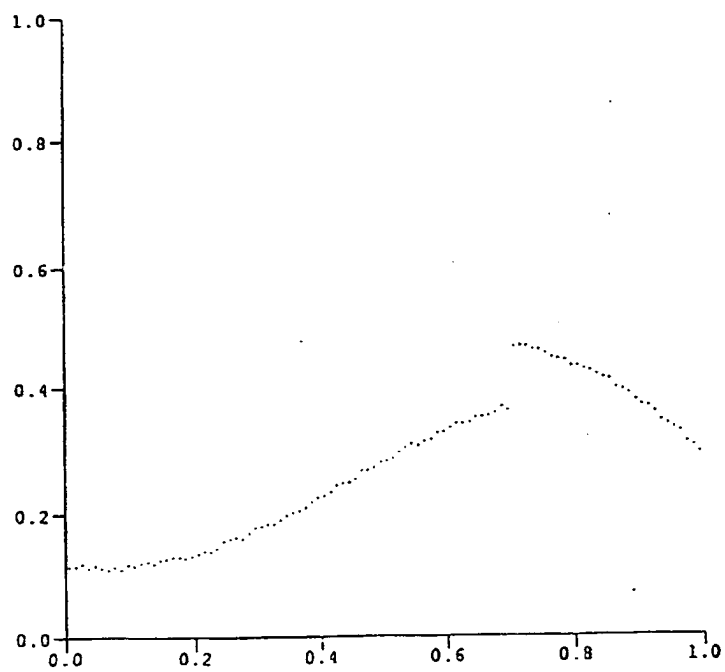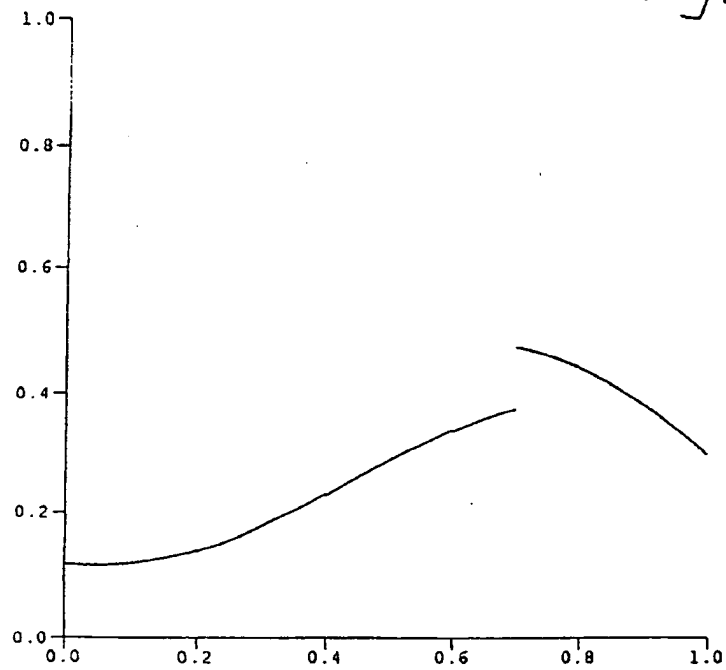
Fig. 5

Fig. 6a

Fig. 66

Fig. 16

Fig. 1a

Fig. 3d

Fig. 2a

Fig. 3c

Fig. 4b

Fig. 4a

Fig. 5

Fig. 6a

Fig. 66